

UNIT V



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Plasma physics:

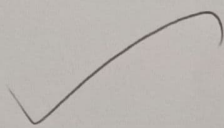
5.1 Definition of plasma

Plasma is sometimes called "the fourth state of matter", beyond the familiar three—solid, liquid and gas. It is a gas in which atoms have been broken up into free-floating negative electrons and positive ions, atoms which have lost electrons and are left with a positive electric charge.

Plasma is a form of matter in which many of the electrons wander around freely among the nuclei of the atoms. Plasma has been called the fourth state of matter, the other three being solid, liquid, and gas.

Normally, the electrons in a solid, liquid, or gaseous sample of matter stay with the same atomic nucleus. Some electrons can move from atom to atom if an electrical current flows in a solid or liquid, but the motion occurs as short jumps by individual electrons between adjacent nuclei. In a plasma, a significant number of electrons have such high energy levels that no nucleus can hold them.

An atom that has lost some of its electrons, thereby attaining an electric charge, is an ion. When a gas is subjected to heat or an electric field, some of its atoms become ions, and the gas is said to be ionized. An ionized gas, unlike a gas in its normal condition, can conduct electrical current to a limited extent. If the heat or electric field becomes extreme, many of the atoms become ions. The resulting super-ionized gas is a plasma, which can conduct a large and sustained electric current.



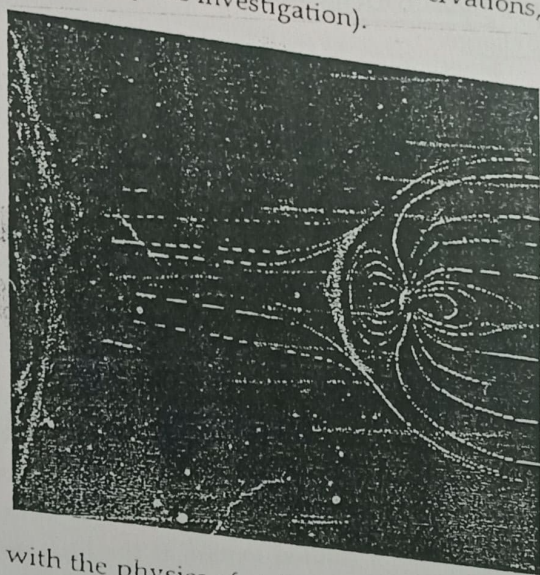
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The behavior and properties of plasmas have aroused interest and creative work among scientists and engineers. Applications include electric lamps, lasers, medical devices, energy converters, water purifiers, and flat-panel video displays (see plasma display). About 99% of the visible universe is formed of plasma

5.2 Its occurrence in Nature

It is estimated that more than 99 percent of the visible universe is made up of plasmas. Plasmas dominate the environment of earth in the solar system and farther out in the interstellar space. Some natural plasma can be found close to the earth's surface. Space physics is the study of the earth environment in space. It is an ambient dominated by many different kinds of plasmas as exemplified in the following list:

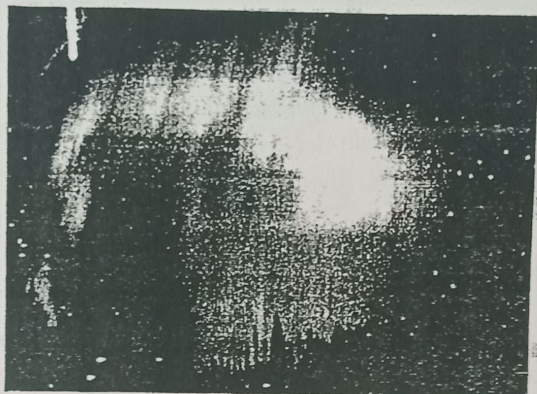
- ✓ Solar wind - Designates the flux of charged particles emitted by the strong activity in the sun. When these particles impinge the magnetosphere (a region in which the configuration of the earth's magnetic field lines remains relatively constant) they are stopped and deflected, creating a shock wave structure which is filled with plasma.
- ✓ Ionosphere - A plasma weakly ionized by the radiation from the sun, which extends from an altitude of 50 km to 10 earth radii with variable density and temperature (10^9 to 10^{12} charged particles/m³, 10² to 10³ K). Van Allen radiation belts - High-energy particles trapped in the magnetic field of the earth (discovered by James Van Allen in 1958, from the earlier earth satellite observations, they indicate the beginning of modern space physics investigation).



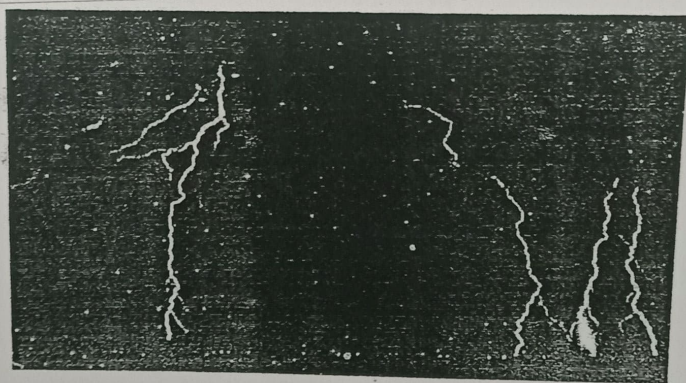
Astrophysics deals with the physics of stellar phenomena that are rich with plasmas under extreme conditions. A few examples of astrophysical plasmas are given below.

Natural plasmas close to earth

Close to the earth surface the occurrence of plasmas is very limited. Life may exist only in much less than 1 percent of the universe in which plasmas do not occur naturally. Two well-known examples of natural plasmas close to earth are below Auroras - Visible glows resulting from the excitation of the atmospheric atoms and molecules, which are bombarded by charged particles that are from the sun and deflected by the geomagnetic field.



Lightning - High current (tens to hundreds of kA) transient electric discharge occurs in the atmosphere with an ordinary extension of a few kilometers. The exact source responsible by the generation of lightning is linked to the electrodynamics of the atmosphere.



Plasma
90% of Universe

The Space Geophysics Division investigates auroras and atmospheric electric phenomena at INPE

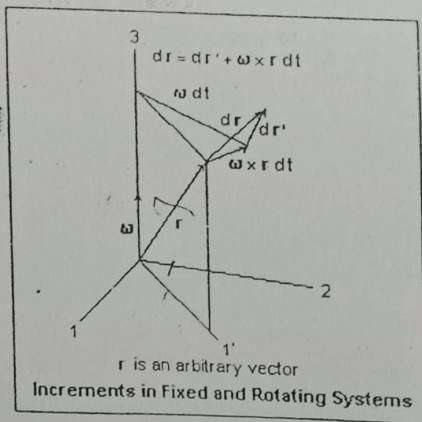
5.3 Dilute and dense plasma

In the past two decades a succession of direct observations by satellites, and extensive computer simulations, has led to the realization that the polar ionosphere

plays a principal role in large-scale magnetospheric processes - a manifestation of the physics linkage involved in solar-terrestrial interactions. Spatial/temporal variations in high-latitude electromagnetic phenomena, such as dynamic aurorae, electric fields and currents, have proved to be extremely complex. Now the challenge is to comprehend the vast amount of complicated measurements made in this magnetosphere-ionosphere system of the Earth. This book addresses the electrical coupling between the hot, but dilute, magnetospheric plasma and the cold, but dense, plasma in the ionosphere. In five major chapters, this book presents: - basic properties of magnetosphere-ionosphere coupling; morphology of electric fields and currents at high latitudes; global modeling of magnetosphere-ionosphere coupling; modeling of ionospheric electrodynamics; current issues, such as auroral particle acceleration, substorms, penetration of high-latitude fields into low latitudes.

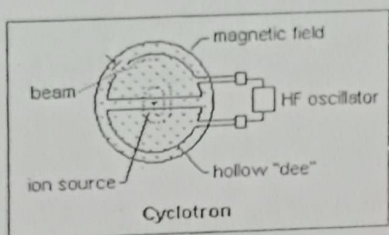
5.4. Uniform but time-dependent magnetic field

We see from the Lorentz equation that the force exerted by the magnetic field on a particle is always normal to its velocity. Therefore, the magnetic field cannot increase the energy of the particle, and so energy is conserved during movement in a purely magnetic, time-independent field. This holds whether the field is uniform or not. We also note that $\text{div } B = 0$ as well, which puts restrictions on the ways the field can vary in space. In the present case, this is satisfied a fortiori, as well as $\text{curl } B = 0$. Incidentally, the field B does not contain any part due to the motion of the particle itself, just as the electric field in the previous section did not contain a contribution from the field of the particle itself.

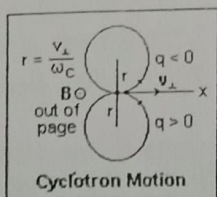


Newton's Law for this problem is $dv/dt = (q/mc)v \times B$. A suggestion of how to solve the problem is shown in the figure at the right. If we re-write the equation of motion $dv/dt - \omega \times v = 0$, where $\omega = -qB/mc$, the left-hand side is the time rate of change of v in a coordinate system rotating with angular velocity ω , so the equation says that this time rate of change is zero. That is, the velocity v' in the rotating system is a

constant. Nothing could be simpler than this! If v_0 is resolved into components parallel and perpendicular to B , then the parallel component is not changed by the rotation and is constant. If we choose the axis so that the rotational velocity about it is equal and opposite to the perpendicular component of the initial velocity in the nonrotating system, then this point remains fixed in the rotating system, and rotates about the axis at a fixed distance r determined by $r\omega = v_{0,perp}$. In general, then, the particle moves on a helix of radius r and pitch $2\pi v_{0,par}/\omega$ in a uniform magnetic field. A positive charge rotates clockwise as seen facing the point of the magnetic field arrow, or anticlockwise riding on the particle looking forward.



The angular frequency $\omega_c = |qB/mc|$ is called the cyclotron frequency. It is the product, in radians per second, of the charge in emu and the magnetic field in gauss. The period of one gyration about the line of force is $T = 2\pi/\omega$. An electron, in a field of one gauss, has a cyclotron frequency of 1.759×10^7 radians per second, or 2.80 MHz. When quoted in herz, the straight frequency is always meant. A proton, in a field of 5000 gauss, has a cyclotron frequency of 47.9 MHz. A 10 MeV proton has a speed of 4.38×10^9 cm/s, so its radius of gyration is 249 cm. A cyclotron, as shown in the diagram at the left, with dees about 5 m in diameter, in a field of 5000 gauss, with an oscillator of frequency 47.9 MHz, could be used to accelerate protons to this energy. The protons would be injected at low velocity at the center of the dees by ionizing hydrogen there, and could be extracted at 10 MeV at the periphery of the dees, after spiraling out, receiving two kicks of, say 500 V, per revolution. Since the angular velocity is constant and independent of the particle velocity, the particles cross the gap between the dees at constant intervals. They would make 10,000 revolutions while being accelerated. A fixed-frequency cyclotron produces a continuous beam, but is limited to non-relativistic particles. An FM (frequency-modulated) cyclotron, or synchrotron decreases the frequency as the mass of the particles increases.



Cyclotron motion, or gyration around the magnetic field, is illustrated in the diagram at the right. The same initial velocity gives loops below for a positive particle, in the direction $v \times B$, and above for a negative particle. The kinetic energy of the cyclotron motion is $U = m v_{o,perp}^2 / 2 = m (\omega r)^2 / 2$. If the particle moves slowly relative to the cyclotron frequency, so that many rotations are made in the time taken to move one radius r , or adiabatically, this motion is very little disturbed, except that the radius and cyclotron frequency may change as the particle moves into regions of different magnetic field. The total kinetic energy of the particle, composed of the energy of gyration and the energy in the direction of the field, remains constant as the particle drifts.

The motion of the charge q represents an average current of q/T around the particle orbit. Since the area of the orbit is $S = \pi r^2$, the gyrotory motion has a magnetic moment $\mu = (i/c)S = q^2 r^2 B / 2mc^2$.

time-dependent magnetic field

If the magnetic field varies with time, an electrical field is produced that is described by Faraday's Law, $\text{curl } E = -(1/c)\partial B/\partial t$. If we integrate this around the gyrotory orbit, the average electric field E along the orbit is given by $2\pi r E = (\pi r^2/c)dB/dt$, or $E = (r/2c)dB/dt$. This is permissible if the magnetic field does not change greatly during one gyration, again the adiabatic condition. The equation of motion is $qE = m dv/dt = m(d/dt)(qBr/mc)$, or $(qr/2c)dB/dt = (d/dt)(qBr/c)$. Then, $(r/2)dB/dt = d(Br)/dt = r dB/dt + B dr/dt$. Therefore, $0 = (r/2)dB/dt + B dr/dt$. Multiplying through by r , we get $d(r^2 B)/dt = 0$. or $r^2 B = \text{constant}$. If this is multiplied by π , we see that this means that the flux linked with the orbit Φ is a constant, and also that the magnetic moment is a constant. This is a very important result, that allows us to find the radius of the orbit at any point as the particle moves, and combined with the conservation of energy, how fast the orbit moves in the direction perpendicular to itself.

This behavior, that a change of magnetic field induces a change in the current so that the flux linkage does not change, is called diamagnetism. Since the magnetic moment is a constant, we can also find the forces acting as if on the center of gyration by the gradient of the magnetic energy, $F = -\text{grad}(\mu B)$. B and μ are, of course, in opposite directions. A negative charge gyrates in the opposite sense to a positive charge, remember. This gives a force in the z -direction, the direction of the magnetic field, of $-\mu(dB/dz)$. However, before we discuss nonuniform magnetic fields, we must discuss combinations of electric and magnetic fields.

5.5. Magnetic pumping

Plasma heating by coalitional magnetic pumping is investigated theoretically. This treatment yields solutions to the energy transfer equations in the form of an energy increase rate, which gives quantitatively the amount of energy increase per rf

driving cycle. The energy increase rates (or heating rates) proportional to the first and second powers of the field modulation factor δ (defined as the ratio of the change in the magnitude of the magnetic field to its background dc value) are derived for an arbitrary rf waveform of the pumping magnetic field. Special cases are examined, including the sinusoidal and sawtooth pumping waveforms. The energy increase rate in the case of a sawtooth waveform was found to be proportional to the first power of δ (first-order heating). This heating rate is many orders of magnitude larger than heating for the sinusoidal case: The latter is proportional to the square of δ and is strongly dependent on the collisionality of the plasma. The use of a sawtooth pumping waveform improves the efficiency of collision magnetic pumping and heating rates comparable to those possible with ion or electron cyclotron resonance heating methods may be achieved.

5.6. Static Non-Uniform Magnetic Fields

Magnetic fields. The experiments utilized a long and narrow yttrium iron garnet film strip magnetized with a spatially non-uniform magnetic field parallel to the strip axis. A microstrip transducer was used to excite spin wave pulses while inductive probe imaging techniques [3] were used to map the spatial evolution of spin waves during propagation. In a non-uniform magnetic field, the wavelength of the spin wave carrier changes while the frequency remains constant. Specifically, the wavelength increases in a spatially decreasing field and decreases in a spatially increasing field, see Figure 1. This response is associated with the spatial changes of the spin wave dispersion response which results in a wave number and velocity change of the propagating pulses.

II.a Experiment Hardware Design For this project many different types of work were required including mechanical design, machine shop metal work, computer modeling, and programming. The mechanical design and machine work was required in order to fabricate a structure which we could use to study spin wave propagation in spatially non-uniform magnetic fields. The design work was done in Turbo Cad. The design was made such that probing devices could be placed near the sample surface in order to probe spin wave excitations, as well as measure magnetic field distributions. The structure designed consists of a stage, and two arms which house (a) Structure 3d view (b) Structure Top View Figure 3: 3D and top views of the structure in Turbo Cad. 4 (a) Arm dimensions (b) Stage dimensions Figure 4: Structure dimensions. 5 Figure 5: Finished Structure two micro strip line transducers/pads which connect to two SMA ports for input/output of microwaves. The SMA ports are used to couple microwave radiation to the transducers which in turn will excite spin waves in our YIG sample.

A schematic created in turbo cad is shown in Figure 3. The dimensions of the structure are shown in Figure 4. The Stage and arms of the structure is composed of scrap aluminum (no-cost) and obtained from the physics machine shop. The

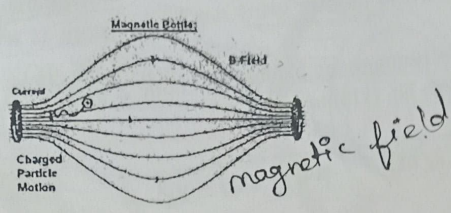
transducer pads were obtained in the lab (no-cost), and the SMA ports were purchased from Coaxicom for P/N: 3215-5CC-1 \$9.99 ea. The assembly of the structure was done via holes tapped for 2-56x3/16 with and non-magnetic socket head screws from Fastener Express P/N ASHC0203. The structure was fabricated using non-magnetic parts. A picture of the finished structure is shown in Figure 5. This structure allows for transducer

input and output, and enables one to position probing mechanisms near the sample surface. The system used for in this project consists of a three axis positioning system (OWIS), and several types of hardware including an oscilloscope (Agilent DSO 81204A -12GHz), Microwave source (HP 83623B), Pulse Generator (SRS DG535), Spectrum Analyzer (Agilent 64440). For this project positioning and data acquisition software was needed in order to position probing mechanisms near the sample surface and record data.

The two probing mechanisms include a custom inductive loop probe for leakage field detection near the YIGs ample surface, as well a standard Magnetic field Hall probe for measuring the distribution. Also numerical computation software was required to decode the acquired data. Positioning and data acquisition software were written in Lab view. Two different programs were required for the two types of data acquisition. One scanning program was written to scan the leakage fields near the sample surface. A second program was written to measure the field distribution near the sample with a Hall probe. In Both cases the probes are mounted on the three axis positioning system. Scans involved both 2-D spatial scans, and 1-D spatial scans. In all cases data acquisition was archived through the use of GPIB connections to desired instruments. Field Distributions were measured using the Lab view program and a FW Bell 9500 Gauss meter. All measurements for the first experiment were preformed with the inductive loop probe. Figure 7: Field Distribution Scanner Data computation software was written in MATLAB. Once data files were obtained from the instrumentation MATLAB was used to extract important features from the data. Figure 6 and Figure 7 are screen shots of the inductive probe scanner program for detectin leakage fields, and field scanner programs form measuring the field distributions

5.7. Magnetic bottle and loss cone

Magnetic bottle
Ass



This image shows how a charge particle will corkscrew along the magnetic fields inside a magnetic bottle, which is two magnetic mirrors placed close together. The particle can be reflected from the dense field region and will be trapped.

A magnetic bottle is two magnetic mirrors placed closed together. For example, two parallel coils separated by a small distance, carrying the same current in the same direction will produce a magnetic bottle between them. Unlike the full mirror machine which typically had many large rings of current surrounding the middle of the magnetic field, the bottle typically has just two rings of current. Particles near either end of the bottle experience a magnetic force towards the center of the region; particles with appropriate speeds spiral repeatedly from one end of the region to the other and back. Magnetic bottles can be used to temporarily trap charged particles. It is easier to trap electrons than ions, because electrons are so much lighter. This technique is used to confine very hot plasmas with temperatures of the order of 106 K.

In a similar way, the Earth's non-uniform magnetic field traps charged particles coming from the sun in doughnut shaped regions around the earth called the "Van Allen radiation belts", which were discovered in 1958 using data obtained by instruments aboard the Explorer 1 satellite.

Magnetic loss

Consider the important case in which the electromagnetic fields do not vary in time. It immediately follows from

$$\frac{d\varepsilon}{dt} = 0, \quad \frac{d\varepsilon}{dt} = 0, \quad \varepsilon = k + e\phi = \frac{m}{2} (v_a^2 + v_E^2) + MB + e\phi$$

$$\frac{d\varepsilon}{dt} = 0, \quad \varepsilon = k + e\phi = \frac{m}{2} (v_a^2 + v_E^2) + MB + e\phi$$

is the total particle energy, and ϕ is the electrostatic potential. Not surprisingly, a charged particle neither gains nor loses energy as it moves around in non-time-varying electromagnetic fields. Since both \mathcal{E} and μ are constants of the motion, we can rearrange Eq. (115) to give

$$U_{\parallel} = \pm \sqrt{(2/m)[\mathcal{E} - \mu B - e\phi] - v_{E}^2}.$$

Thus, in regions where $\mathcal{E} > \mu B + e\phi + m v_{E}^2/2$ charged particles can drift in either direction along magnetic field-lines. However, particles are excluded from

$$\mathcal{E} < \mu B + e\phi + m v_{E}^2/2$$

regions where (since particles cannot have imaginary parallel velocities!). Evidently, charged particles must reverse direction at those

$$\mathcal{E} = \mu B + e\phi + m v_{E}^2/2$$

points on magnetic field-lines where such points are termed "bounce points" or "mirror points." Let us now consider how we might construct a device to confine a collisionless (i.e., very hot) plasma. Obviously, we cannot use conventional solid walls, because they would melt. However, it is possible to confine a hot plasma using a magnetic field (fortunately, magnetic fields do not melt!); this technique is called magnetic confinement. The electric field in

$$\mathbf{E} \ll B\mathbf{v} \quad \mathbf{E} \times \mathbf{B}$$

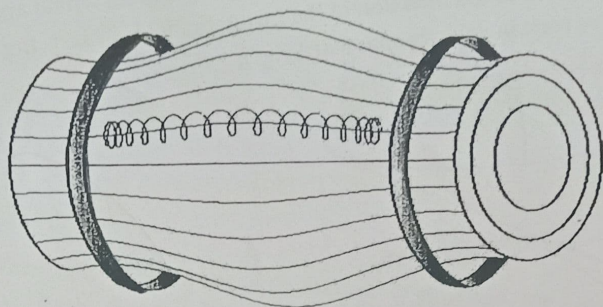
confined plasmas is usually weak (i.e., $\mathbf{E} \ll B\mathbf{v}$), so that the drift is similar in magnitude to the magnetic and curvature drifts. In this case, the bounce point

$$U_{\parallel} = 0$$

condition, $U_{\parallel} = 0$, reduces to

$$\mathcal{E} = \mu B.$$

Consider the magnetic field configuration shown in Fig. 1. This is most easily produced using two Helmholtz coils. Incidentally, this type of magnetic confinement device is called a magnetic mirror machine. The magnetic field configuration obviously possesses axial symmetry. Let z be a coordinate which measures distance along the axis of symmetry. Suppose that $z = 0$ corresponds to the mid-plane of the device (i.e., halfway between the two field coils).



It is clear from Fig. 1 that the magnetic field-strength $B(z)$ on a magnetic field-line situated close to the axis of the device attains a local minimum B_{\min} at $z = 0$ and increases symmetrically as $|z|$ increases until reaching a maximum value B_{\max} about the location of the two field-coils, and then decreases as $|z|$ is further increased. According to (1), any particle which satisfies the inequality

$$\mu > \mu_{\text{trap}} = \frac{\mathcal{E}}{B_{\max}}$$

is trapped on such a field-line. In fact, the particle undergoes periodic motion along the field-line between two symmetrically placed mirror points. The magnetic field strength at the mirror points

$$B_{\text{mirror}} = \frac{\mu_{\text{trap}}}{\mu} B_{\max} < B_{\max}.$$

Now, on the mid-plane $\mu = m v_{\perp}^2 / 2 B_{\min}$ and $\mathcal{E} = m (v_{\parallel}^2 + v_{\perp}^2) / 2$ (n.b. From now on, we shall write $\mathbf{v} = v_{\parallel} \mathbf{b} + \mathbf{v}_{\perp}$, for ease of notation.) Thus, the trapping condition reduces to

$$\frac{|v_{\parallel}|}{|v_{\perp}|} < (B_{\max} / B_{\min} - 1)^{1/2}.$$

Particles on the mid-plane which satisfy this inequality are trapped: particles which do not satisfy this inequality escape along magnetic field-lines. Clearly, a magnet

mirror machine is incapable of trapping charged particles which are moving parallel, or nearly parallel, to the direction of the magnetic field. In fact, the above inequality defines a loss cone in velocity space—see Fig. 2.

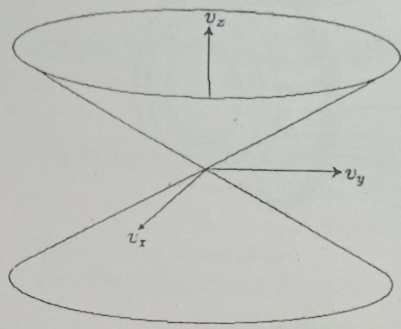


Figure 2: Loss cone in velocity space. The particles lying inside the cone are not reflected by the magnetic field.

It is clear that if plasma is placed inside a magnetic mirror machine then all of the particles whose velocities lie in the loss cone promptly escape, but the remaining particles are confined. Unfortunately, that is not the end of the story. There is no such thing as an absolutely collisionless plasma. Collisions take place at a low rate even in very hot plasmas. One important effect of collisions is to cause diffusion of particles in velocity space. Thus, in a mirror machine collisions continuously scatter trapped particles into the loss cone, giving rise to a slow leakage of plasma out of the device. Even worse, plasmas whose distribution functions deviate strongly from an isotropic Maxwellian (e.g., a plasma confined in a mirror machine) are prone to velocity space instabilities, which tend to relax the distribution function back to a Maxwellian. Clearly, such instabilities are likely to have a disastrous effect on plasma confinement in a mirror machine. For these reasons, magnetic mirror machines are not particularly successful plasma confinement devices, and attempts to achieve nuclear fusion using this type of device have mostly been abandoned.

5.8. MHD Equation

Magneto hydrodynamics (MHD) (magneto fluid dynamics or hydromagnetics) is the study of the dynamics of electrically conducting fluids. Examples of such fluids include plasmas, liquid metals, and salt water or electrolytes. The word magneto hydrodynamics (MHD) is derived from magneto- meaning magnetic field, hydro- meaning liquid, and -dynamics meaning movement. The field of MHD was initiated by Hannes Alfvén, for which he received the Nobel Prize in Physics in 1970.

Text

The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and also changes the magnetic field itself. The set of equations which describe MHD are a combination

of heavier of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations have to be solved simultaneously, either analytically or numerically.

Ideal MHD equations

The ideal MHD equations consist of the continuity equation, the Cauchy momentum equation, Ampere's Law neglecting displacement current, and a temperature evolution equation. As with any fluid description to a kinetic system, a closure approximation must be applied to highest moment of the particle distribution equation. This is often accomplished with approximations to the heat flux through a condition of adiabaticity or isothermality.

In the following, \mathbf{B} is the magnetic field, \mathbf{E} is the electric field, \mathbf{V} is the bulk plasma velocity, \mathbf{J} is the current density, ρ is the mass density, p is the plasma pressure, and t is time. The continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0.$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

The momentum equation is

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = \mathbf{J} \times \mathbf{B} - \nabla p.$$

$$\rho \left(\frac{\partial}{\partial t} + \nabla \cdot \mathbf{V} \right) \mathbf{V} = \mathbf{J} \times \mathbf{B}$$

The Lorentz force term $\mathbf{J} \times \mathbf{B}$ can be expanded to give

$$\mathbf{J} \times \mathbf{B} = \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{\mu_0} - \nabla \left(\frac{B^2}{2\mu_0} \right),$$

where the first term on the right hand side is the magnetic tension force and the second term is the magnetic pressure force. The ideal Ohm's law for a plasma is given by

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = 0.$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = 0$$

Faraday's law is

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}.$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

The low-frequency Ampere's law neglects displacement current and is given by

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}.$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

The magnetic divergence constraint is

$$\nabla \cdot \mathbf{B} = 0.$$

The energy equation is given by

$$\frac{d}{dt} \left(\frac{p}{\rho^\gamma} \right) = 0,$$

where $\gamma = 5/3$ is the ratio of specific heats for an adiabatic equation of state. This energy equation is, of course, only applicable in the absence of shocks or heat conduction as it assumes that the entropy of a fluid element does not change.

5.9. Magnetic Reynolds number

The Magnetic Reynolds number is a dimensionless group that occurs in magneto hydrodynamics. It gives an estimate of the effects of magnetic advection to magnetic diffusion, and is typically defined by:

$$R_m = \frac{UL}{\eta} \sim$$

$$R_m = \frac{\text{induction}}{\text{diffusion}}$$

where

U is a typical velocity scale of the flow

L is a typical length scale of the flow

η is the magnetic diffusivity

For $R_m \ll 1$, advection is relatively unimportant, and so the magnetic field will tend to relax towards a purely diffusive state, determined by the boundary conditions rather than the flow.

For $R_m \gg 1$, diffusion is relatively unimportant on the length scale L . Flux lines of the magnetic field are then advected with the fluid flow, until such time as gradients are concentrated into regions of short enough length scale that diffusion can balance advection.

5.10. Pinched plasma

A pinch is the compression of an electrically conducting filament by magnetic forces. The conductor is usually a plasma, but could also be a solid or liquid metal. In a z-pinch, the current is axial (in the z direction in a cylindrical coordinate system) and the magnetic field azimuthal, in a theta-pinch, the current is azimuthal (in the theta direction in cylindrical coordinates) and the magnetic field is axial. The phenomenon may also be referred to as a "Bennett pinch" (after Willard Harrison Bennett), "electromagnetic pinch", "magnetic pinch", "pinch effect" or "plasma pinch"

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Pinches occur naturally in electrical discharges such as lightning bolts, theaurora, current sheets, and solar flares. They are also produced in the laboratory, primarily for research into fusion power, but also by hobbyists (crushing aluminums cans). [citation needed] section of the crushed lightning rod studied by Pollock and Barraclough. The rod is in the collection of the School of Physics, Sydney, Australia.

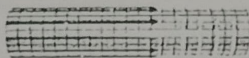
Pinches are created in the laboratory in equipment related to nuclear fusion, such as the Z-pinch machine, and high-energy physics, such as the dense plasma focus. Pinches may also become unstable, and generate radiation across the electromagnetic spectrum, including radio waves, x-rays and gamma rays, and also neutrons and synchrotron radiation. Types of pinches, that may differ in geometry and operating forces, include the cylindrical pinch, inverse pinch, orthogonal pinch effect, reversed field pinch, sheet pinch, screw pinch (also called stabilized z-pinch, or theta-z pinch), theta pinch (or thetatron), toroidal pinch, ware pinch and Z-pinch.

Pinches are used to generate X-rays, and the intense magnetic fields generated are used in electromagnetic forming of metals (they have been demonstrated in crushing aluminums soft drinks cans). They have applications to particle beams including particle beam weapons, and astrophysics.

One-dimensional configurations

There are three analytic one dimensional configurations generally studied in plasma physics. These are the theta-pinch, the Z-pinch, and the Screw Pinch. All of the classic one dimensional pinches are cylindrically shaped. Symmetry is assumed in the axial (z) direction and in the azimuthal (theta) direction. It is traditional to name a one-dimensional pinch after the direction in which the current travels.

The θ -pinch



The θ -pinch has a magnetic field traveling in the z direction. Using Ampère's law (discarding the displacement term)

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \times B = \mu_0 J$$

$$\vec{B} = B_z(r)\hat{z}$$

$$\mu_0 \vec{J} = \frac{1}{r} \frac{d}{d\theta} B_z \hat{r} - \frac{d}{dr} B_z \hat{\theta}$$

Since B is only a function of r we can simplify this to

$$\mu_0 \vec{J} = -\frac{d}{dr} B_z \hat{\theta}$$

So J points in the θ direction. θ -pinches tend to be resistant to plasma instabilities. This is due in part to the frozen in flux theorem, which is beyond the scope of this article.

The Z-Pinch has a magnetic field in the θ direction. Again, by electrostatic Ampere's Law

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{B} = B_\theta(r)\hat{\theta}$$

$$\mu_0 \vec{J} = \frac{1}{r} \frac{d}{dr} (r B_\theta) \hat{z} - \frac{d}{dz} B_\theta \hat{r}$$

$$\mu_0 \vec{J} = \frac{1}{r} \frac{d}{dr} (r B_\theta) \hat{z}$$

So J points in the z direction. Since particles in a plasma basically follow magnetic field lines, Z-pinches lead them around in circles. Therefore, they tend to have excellent confinement properties.

The screw pinch The screw pinch is an effort to combine the stability aspects of the θ -pinch and the confinement aspects of the Z-pinch. Referring once again to Ampere's Law

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

But this time, the B field has a θ component and a z component

$$\vec{B} = B_\theta(r)\hat{\theta} + B_z(r)\hat{z}$$

$$\mu_0 \vec{J} = \frac{1}{r} \frac{d}{dr} (r B_\theta) \hat{z} - \frac{d}{dr} B_z \hat{\theta}$$

So this time J has a component in the z direction and a component in the θ direction.

Two-dimensional equilibria



A toroidal coordinate system in common use in plasma physics. The red arrow indicates the poloidal direction (θ) and the blue arrow indicates the toroidal direction (φ)

A common problem with one-dimensional equilibria based machines is end losses. As mentioned above, most of the motion of particles in a plasma is directed along the magnetic field. With the θ -pinch and the screw-pinch, this leads particles to the end of the machine very quickly (as the particles are typically moving quite fast). Additionally, the Z-pinch has major stability problems. Though particles can be reflected to some extent with magnetic mirrors, even these allow many particles to pass. The most common method of mitigating this effect is to bend the cylinder around into a torus. Unfortunately this breaks θ symmetry, as paths on the inner portion (inboard side) of the torus are shorter than similar paths on the outer portion (outboard side). Thus, a new theory is needed. This gives rise to the famous Grad-Shafranov equation.

The one dimensional equilibria provide the inspiration for some of the toroidal configurations. An example of this is the ZETA device at Culham England (which also operated as a Reversed Field Pinch). The most well recognized of these devices is the toroidal version of the screw pinch, the Tokamak.

Numerical solutions to the Grad-Shafranov equation have also yielded some equilibria, most notably that of the reversed field pinch.

Three-dimensional equilibria

There does not exist a coherent analytical theory for three-dimensional equilibria. The general approach to finding three dimensional equilibria is to solve the vacuum ideal MHD equations. Numerical solutions have yielded designs for stellarators. Some machines take advantage of simplification techniques such as helical symmetry (for example University of Wisconsin's Helically Symmetric eXperiment).

5.11. The Bennett relation

Consider a cylindrical column of fully ionized quasineutral plasma, with an axial electric field, producing an axial current density, j_z and associated azimuthal magnetic field, B_ϕ . As the current flows through its own magnetic field, a pinch is generated with an inward radial force density of $j \times B$. In a steady state with forces balancing:

$\nabla p = \nabla(p_e + p_i) = j \times B$

$\nabla p = \nabla(p_e + p_i)$
 $j \times B$

where ∇p is the magnetic pressure gradient, p_e and p_i is the electron and ion pressures. Then using Maxwell's equation $\nabla \times B = \mu_0 j$ and the ideal gas law $p = N k T$, we derive:

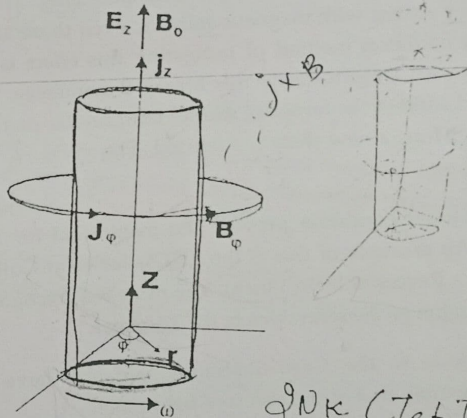
$2Nk(T_e + T_i) = \frac{\mu_0}{4\pi} I^2$

μ_0
 $p = nkT$

$p = nkT$

the Bennett relation)

where N is the number of electrons per unit length along the axis, T_e and T_i are the electron and ion temperatures, I is the total beam current, and



k is the Boltzmann constant.

$2Nk(T_e + T_i)$

The generalized Bennett relation

The generalized Bennett relation considers a current-carrying magnetic-field-aligned cylindrical plasma pinch undergoing rotation at angular frequency ω

The Generalized Bennett Relation considers a current-carrying magnetic-field-aligned cylindrical plasma pinch undergoing rotation at angular frequency ω . Along the axis of the plasma cylinder flows a current density j_z , resulting in an azimuthal

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magnetic field B_ϕ . Originally derived by Witalis, the Generalized Bennett Relation results in:

$$\left(\frac{1}{4} \frac{\partial^2 J_0}{\partial t^2} = W_{\perp \text{kin}} + \Delta W_{E_r} + \Delta W_{E_z} + \Delta W_k - \frac{\mu_0}{8\pi} I^2(a) - \frac{1}{2} Gm^2 N^2(a) + \frac{1}{2} \pi a^2 \epsilon_0 (E_r^2(a) - E_\phi^2(a)) \right)$$

$\frac{1}{2} \frac{\partial^2 J_0}{\partial t^2} = k^2 J_0 \dots$

where a current-carrying, magnetic-field-aligned cylindrical plasma has a radius a , J_0 is the total moment of inertia with respect to the z axis, $W_{\perp \text{kin}}$ is the kinetic energy per unit length due to beam motion transverse to the beam axis, ΔW_{B_z} is the self-consistent B_z energy per unit length, ΔW_{E_z} is the self-consistent E_z energy per unit length, W_k is thermokinetic energy per unit length, a is the axial current inside the radius a (r in diagram), $N(a)$ is the total number of particles per unit length, E_r is the radial electric field, E_ϕ is the rotational electric field.

The positive terms in the equation are expansional forces while the negative terms represent beam compressional forces.

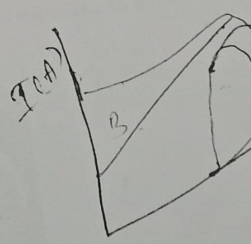
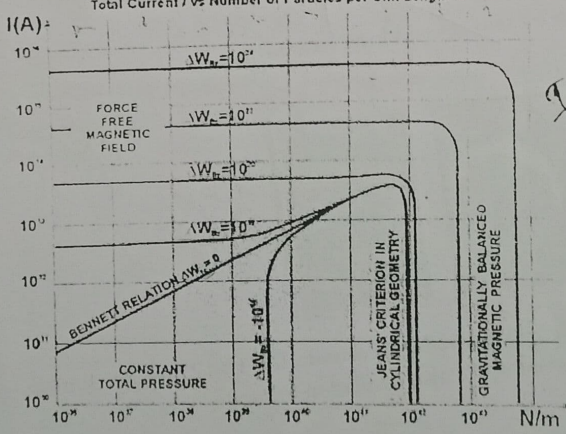
The Carlqvist relation

The Carlqvist Relation, published by Per Carlqvist in 1988, is a specialization of the Generalized Bennett Relation (above), for the case that the kinetic pressure is much smaller at the border of the pinch than in the inner parts. It takes the form

$$\frac{\mu_0}{8\pi} I^2(a) + \frac{1}{2} Gm^2 N^2(a) = \Delta W_{B_z} + \Delta W_k$$

and is applicable to many space plasmas.

BENNETT PINCH
Total Current I vs Number of Particles per Unit Length N



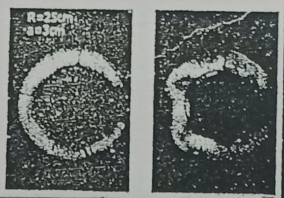
The Bennett pinch showing the total current (I) versus the number of particles per unit length (N). The chart illustrates four physically distinct regions. The plasma temperature is 20 K, the mean particle mass 3×10^{-27} kg, and ΔW_{Bz} is the excess magnetic energy per unit length due to the axial magnetic field B_z . The plasma is assumed to be non-rotational, and the kinetic pressure at the edges is much smaller than inside.

The Carlqvist Relation can be illustrated (see right), showing the total current (I) versus the number of particles per unit length (N) in a Bennett pinch. The chart illustrates four physically distinct regions. The plasma temperature is quite cold ($T_i = T_e = T_n = 20$ K), containing mainly hydrogen with a mean particle mass 3×10^{-27} kg. The thermokinetic energy $W_k \gg na^2 p_k(a)$. The curves, ΔW_{Bz} show different amounts of excess magnetic energy per unit length due to the axial magnetic field B_z . The plasma is assumed to be non-rotational, and the kinetic pressure at the edges is much smaller than inside.

Chart regions: (a) In the top-left region, the pinching force dominates. (b) Towards the bottom, outward kinetic pressures balance inwards magnetic pressure, and the total pressure is constant. (c) To the right of the vertical line $\Delta W_{Bz} = 0$, the magnetic pressures balances the gravitational pressure, and the pinching force is negligible. (d) To the left of the sloping curve $\Delta W_{Bz} = 0$, the gravitational force is negligible. Note that the chart shows a special case of the Carlqvist relation, and if it is replaced by the more general Bennett relation, then the designated regions of the chart are not valid.

Carlqvist further notes that by using the relations above, and a derivative, it is possible to describe the Bennett pinch, the Jeans criterion (for gravitational instability, in one and two dimensions), force-free magnetic fields, gravitationally balanced magnetic pressures, and continuous transitions between these states.

5.12. Qualitative discussion on sausage and kink instability



$$\frac{\partial p}{\partial z} = \frac{1}{4\pi} \frac{\partial B_z}{\partial z}$$

One of the earliest photos of the kink instability in action - the 3 by 25 cm pyrex tube at Aldermaston. A kink instability, also oscillation or mode, is a class

sausage
kink!

Suzanna

field
MHD

of magneto-hydrodynamic instabilities which sometimes develop in a thin plasma column carrying a strong axial current. If a "kink" begins to develop in a column the magnetic forces on the inside of the kink become larger than those on the outside, which leads to growth of the perturbation. As it develops at fixed areas in the plasma, kinks belong to the class of "absolute plasma instabilities", as opposed to convective processes. The kink instability was first widely explored in the Z-pinch fusion power machines in the 1950s.

The geometrical configuration of the problem is illustrated in . The equation of motion in the framework of incompressible ideal MHD for a nonstationary plasma has the following form,

$$\rho \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] + \nabla P = \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B}, \quad \text{--- (1)}$$

where ρ is the plasma density, \mathbf{V} is the plasma bulk velocity, \mathbf{B} is the magnetic field, and P is the total pressure. Let us consider a plasma element of a unit volume, placed at the center of the current layer. In the equilibrium state, the total pressure gradient compensates the magnetic tension,

$$\frac{\partial P}{\partial z} = -\frac{1}{4\pi} B_x \frac{\partial B_x}{\partial x} \quad \text{--- (2)}$$

A small displacement, δz , of this plasma element along the z-direction yields the restoring force F_z , which is the difference of two forces, caused by the magnetic tension and the total pressure gradient,

$$F_z = -\frac{1}{4\pi} \delta z \left(\frac{\partial B_x}{\partial z} \frac{\partial B_x}{\partial x} \right)_{z=0}, \quad \text{--- (3)}$$

where $B_x(z)$ is determined from a Taylor series expansion. This force accelerates plasma in the z-direction, as shown in Fig. The equation of motion of this plasma element has the form,

$$\frac{\partial^2(\delta z)}{\partial t^2} = -\omega_j^2 \delta z, \quad \text{--- (4)}$$

(4)

where

$$\frac{\partial^2 \delta z}{\partial t^2} = -\omega_j^2 \delta z$$

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$$\omega_f = \sqrt{\frac{1}{4\pi\rho} \frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x}}$$

In the case of a positive product of the two magnetic gradients, the parameter ω is real, and it has the meaning of the characteristic frequency of the flapping wave oscillations. In the opposite case of a negative product of the magnetic gradients, the current sheet is unstable. The flapping perturbations can grow up exponentially without propagation, because ω is pure imaginary. These two cases are characterized by a different behaviour of the background total pressure. Specifically, the total pressure has a maximum at the center of the current sheet for an unstable situation, and it has a minimum for stable conditions.

Poynting theorem

recharge wave gauge

free space E.M Generation

Completed

E. M. T

$$\Omega_s = \sqrt{\frac{1}{4\pi\rho} \frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x}}$$

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