

UNIT IV

ELECTROMAGNETIC WAVES

4.1. Generation of Electromagnetic Waves.

Electromagnetic waves are generated by moving electrons. An electron generates an electric field which we can visualize as lines radiating from the electron. If the electron moves, say it vibrates back and forth, then this motion will be transferred to the field lines and they will become wavy. In turn, the moving electron generates a magnetic field that will also become wavy from the motion of the electron.

These combined electrical and magnetic waves reinforce one another. This kind of wave is called an electromagnetic wave and light is such a wave. Since all matter contains electrons and all these electrons are in motion, as are the atomic nuclei they spin around, all matter generates electromagnetic waves. Since all electromagnetic waves travel at the same speed (c) the frequency of the waves is determined by the frequency of the vibrating electrons that generate them. Hot substances have more energy and their component atoms vibrate more rapidly than those of cold bodies.

Thus the peak energy radiated by hot bodies has a higher frequency, shorter wavelength, than that of cooler bodies. The relationship of the peak frequency of a black body to its absolute temperature is expressed by Wein's Law. $\lambda_{\max} = a/T$
 $a = 2989$ if λ is measured in microns

Electromagnetic (EM) radiation is emitted by all matter and consists of orthogonal electrical and magnetic waves. All EM travels at the same speed through a vacuum (186,000 miles per second or 300,000 kilometers per sec.). This radiation is generated by a moving charge or charges. All matter consists of atoms in motion and these in turn consist of positively charged protons surrounded by a cloud of negatively charged electrons. The vibrating motion of the atoms causes the cloud of electrons to oscillate and this oscillation generates electromagnetic radiation. Since all electromagnetic radiation travels at the same velocity the frequency and wavelength of the generated radiation depends on the frequency of the oscillating electron cloud.

(Thus, on average, cool objects (say those at room temperature) generate long wavelength (low frequency) radiation, while hot objects (such as the sun) generate short wavelength (high frequency) radiation.)

4.2. Electromagnetic Wave Equation

The wave equation for a plane electric wave traveling in the x direction in space is

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

with the same form applying to the magnetic field wave in a plane perpendicular the electric field. Both the electric field and the field are perpendicular to the direction of travel x . The symbol c represents the speed of other electromagnetic waves. The wave equation for electromagnetic waves arises from Maxwell's equations. The

$$E = E_m \sin(kx - \omega t)$$

form of a plane wave solution for the electric field is

$$B = B_m \sin(kx - \omega t)$$

and that for the magnetic field

To be consistent with Maxwell's equations, these solutions must be related by

$$\frac{E_m}{B_m} = c$$

The magnetic field B is perpendicular to the electric field E in the orientation where the vector product $E \times B$ is in the direction of the propagation of the wave. Energy in Electromagnetic Waves

Electromagnetic waves carry energy as they travel through empty space. There is an energy density associated with both the electric and magnetic fields. The rate of energy transport per unit area is described by the vector

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

which is called the Poynting vector. This expression is a vector product, and since the magnetic field is perpendicular to the electric field, the magnitude can be written

$$S = \frac{1}{\mu_0} EB$$

The rate of energy transport S is perpendicular to both E and B and in the direction of propagation of the wave. A condition of the wave solution for a plane wave is $B_m = E_m/c$ so that the average intensity for a plane wave can be written

$$S = \frac{1}{c\mu_0} E_m^2 \overline{\sin^2(kx - \omega t)} = \frac{1}{c\mu_0} \frac{E_m^2}{2}$$

This makes use of the fact that the average of the square of a sinusoidal function over a whole number of periods is just 1/2.

The electromagnetic wave equation is a second-order partial differential equation that describes the propagation of electromagnetic waves through a medium or in a vacuum. It is a three-dimensional form of the wave equation. The homogeneous form of the equation, written in terms of either the electric field E or the magnetic field B , takes the form:

$$\left(\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2} \right) \mathbf{E} = 0$$

$$\left(\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2} \right) \mathbf{B} = 0$$

where

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

is the speed of light in a medium with permeability (μ), and permittivity (ϵ), and ∇^2 is the Laplace operator. In a vacuum, $c = c_0 = 299,792,458$ meters per second, which is the speed of light in free space. The electromagnetic wave equation derives from Maxwell's equations. It should also be noted that in most older literature, B is called the magnetic flux density or magnetic induction.

The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

Maxwell's derivation of the electromagnetic wave equation has been replaced in modern physics education by a much less cumbersome method involving combining the corrected version of Ampere's circuital law with Faraday's law of induction.

To obtain the electromagnetic wave equation in a vacuum using the modern method, we begin with the modern 'Heaviside' form of Maxwell's equations. In a vacuum- and charge-free space, these equations are:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{B} &= \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{aligned}$$

where $\rho = 0$ because there's no charge density in free space.

Taking the curl of the curl equations gives:

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) &= -\frac{\partial}{\partial t} \nabla \times \mathbf{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \\ \nabla \times (\nabla \times \mathbf{B}) &= \mu_0 \epsilon_0 \frac{\partial}{\partial t} \nabla \times \mathbf{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \end{aligned}$$

We can use the vector identity

$$\nabla \times (\nabla \times \mathbf{V}) = \nabla (\nabla \cdot \mathbf{V}) - \nabla^2 \mathbf{V}$$

where \mathbf{V} is any vector function of space. And

$$\nabla^2 \mathbf{V} = \nabla \cdot (\nabla \mathbf{V})$$

where $\nabla \mathbf{V}$ is a dyadic which when operated on by the divergence operator $\nabla \cdot$ yields a vector. Since

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned}$$

then the first term on the right in the identity vanishes and we obtain the wave

$$\begin{aligned} \frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} &= 0 \\ \frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} &= 0 \end{aligned}$$

where

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792458 \times 10^8 \text{ m/s}$$

is the speed of light in free space. variant form of the homogeneous wave equation

Time dilation in transversal motion. The requirement that the speed of light is constant in every inertial reference frame leads to the Relativity. These relativistic equations can be written in contra variant form as

$$\square A^\mu = 0$$

where the electromagnetic four-potential is

$$A^\mu = (\phi/c, \mathbf{A})$$

with the Lorenz gauge condition:

$$\partial_\mu A^\mu = 0,$$

where

$$-\square = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$

is the d'Alembertian operator. (The square box is not a typographical error; it is the correct symbol for this operator.)

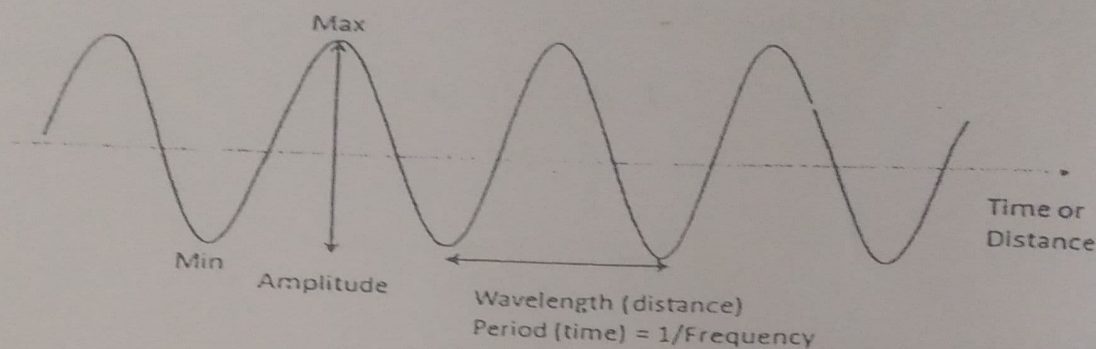
4.3. Wave Parameters

The maximum displacement of the medium in either direction is the amplitude of the wave. The distance between successive crests or successive troughs (corresponding to maximum displacements in the same direction) is the wavelength of the wave. The frequency of the wave is equal to the number of crests (or troughs) that pass a given fixed point per unit of time. Closely related to the frequency is the period of the wave, which is the time lapse between the passage of successive crests (or troughs). The frequency of a wave is the inverse of the period.

One full wavelength of a wave represents one complete cycle, that is, one complete vibration in each direction. The various parts of a cycle are described by the phase of the wave; all waves are referenced to an imaginary synchronous motion in a circle; thus the phase is measured in angular degrees, one complete cycle being 360° . Two waves whose corresponding parts occur at the same time are said to be in phase. If the two waves are at different parts of their cycles, they are out of phase. Waves out of phase by 180° are in phase opposition. The various phase relationships between combining waves determines the type of interference that takes place.

The speed of a wave is determined by its wavelength λ and its frequency ν , according to the equation $v = \lambda\nu$, where v is the speed, or velocity. Since frequency is inversely related to the period T , this equation also takes the form $v = \lambda/T$. The speed of a wave tells how quickly the energy it carries is being transferred. It is important to note that the speed is that of the wave itself and not of the medium through which it is traveling. The medium itself does not move except to oscillate as the wave passes. Interaction of the sound wave with a medium and include pressure, density, temperature, and particle motion.

Wave parameters are the group of characteristics that identify a wave. They are shown in the figure at the below. From these components, one can usually derive other wave properties (i.e. period, power and intensity) based on known equations.



4.4. Velocity

Electromagnetic waves travel through a vacuum at a constant velocity of 2.99792×10^8 m/s, which is known as the speed of light, c. The relationship between the speed of light, wavelength, and frequency is:

$$c = \lambda v$$

$$c = \lambda v$$

When light passes through other media, the velocity of light decreases. Since the energy of a photon is fixed, the frequency of a photon does not change. Thus for a given frequency of light, the wavelength must decrease as the velocity decreases. The decrease in velocity is quantitated by the refractive index, n, which is the ratio of c to the velocity of light in another medium, v:

$$n = c/v$$

$$n = c/v$$

$$c = \lambda v$$

$$n = \frac{c}{v}$$

*measured with 589.3 nm light

4.5. Intrinsic Impedance

The intrinsic impedance is a property of a medium - an area of space. For a vacuum (outer space) or for wave propagation through the air around earth (often called 'free space'), the intrinsic impedance (often written as η or Z) is given by:

$$\eta = Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi 10^{-7}}{8.854 \times 10^{-12}}} \approx 120\pi \approx 377 \text{ Ohms}$$

This parameter is the ratio of the magnitude of the E-field to the magnitude of the H-field for a plane wave in a lossless medium (zero conductivity):

$$Z = \frac{|\mathbf{E}|}{|\mathbf{H}|}$$

This relation can be derived directly from Maxwell's Equations. For a general medium with permittivity and permeability given by $(\epsilon, \mu) = (\epsilon, \epsilon_0 \cdot \mu, \mu_0)$, the intrinsic impedance is given by:

$$Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} = \sqrt{\frac{\mu_r}{\epsilon_r}} Z_0$$

For a medium with a conductivity σ associated with it, the intrinsic impedance is

$$Z = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \quad \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

given by:

When the conductivity is non-zero, the above intrinsic impedance is a complex number, indicating that the electric and magnetic fields are not in-phase. The intrinsic impedance of free-space has nothing to do with the electrical impedance of an antenna. Also, there is no reason to have the impedance of an antenna match the intrinsic impedance of free space (no mismatch loss occurs).

4.6. Propagation constant

"Transmission parameter" redirects here. For ABCD transmission parameters, see two-port ABCD-parameters. For scattering transfer parameters, see scattering parameters#Scattering transfer parameters. The propagation constant of an electromagnetic wave is a measure of the change undergone by the amplitude of the wave, as it propagates in a given direction. The quantity being measured can be the voltage or current in a circuit or a field vector such as electric field strength or flux density. The propagation constant itself measures change per metre but is otherwise dimensionless. In the context of two-port networks and their cascades, propagation constant measures the change undergone by the source quantity as it propagates from one port to the next.

The propagation constant is expressed logarithmically, almost universally to the base e , rather than the more usual base 10 used in telecommunications in other situations. The quantity measured, such as voltage, is expressed as a sinusoidal pharos. The phase of the sinusoid varies with distance which results in the propagation constant being a complex number, the imaginary part being caused by the phase change.

The term propagation constant, is somewhat of a misnomer as it usually varies strongly with ω . It is probably the most widely used term but there are a large variety of alternative names used by various authors for this quantity. These

include, transmission parameter, transmission function, propagation parameter, propagation coefficient and transmission constant. In plural, it is usually implied that α and β are being referenced separately but collectively as in transmission parameters, propagation coefficients, transmission constants and secondary coefficients. This last occurs in transmission line theory, the term secondary being used to contrast to the primary line coefficients. The primary coefficients being the physical properties of the line; R, C, L and G, from which the secondary coefficients may be derived using the telegrapher's equation. Note that, at least in the field of transmission lines, the term transmission coefficient has a different meaning despite the similarity of name. Here it is the corollary of reflection coefficient.

The propagation constant, symbol γ , for a given system is defined by the ratio of the amplitude at the source of the wave to the amplitude at some distance x , such that,

$$\frac{A_0}{A_x} = e^{\gamma x}$$

Since the propagation constant is a complex quantity we can write:

$$\gamma = \alpha + i\beta$$

where

α , the real part, is called the attenuation constant

β , the imaginary part, is called the phase constant

That β does indeed represent phase can be seen from Euler's formula;

$$e^{i\theta} = \cos\theta + i\sin\theta$$

which is a sinusoid which varies in phase as θ varies but does not vary in amplitude because;

$$|e^{i\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

The reason for the use of base e is also now made clear. The imaginary phase constant, $i\beta$, can be added directly to the attenuation constant, α , to form a single complex number that can be handled in one mathematical operation provided they are to the same base. Angles measured in radians require base e , so the attenuation is likewise in base e .

The propagation constant for copper (or any other conductor) lines can be calculated from the primary line coefficients by means of the relationship;

$$\gamma = \sqrt{ZY}$$

where;

$Z = R + i\omega L$, the series impedance of the line per meter and,

$Y = G + i\omega C$, the shunt admittance of the line per meter.

In telecommunications, the term attenuation constant, also called attenuation parameter or attenuation coefficient, is the attenuation of an electromagnetic wave propagating through a medium per unit distance from the source. It is the real part of the propagation constant and is measured in nepers per meter. A neper is approximately 8.7dB. Attenuation constant can be defined by the amplitude ratio;

The propagation constant per unit length is defined as the natural logarithmic of ratio of the sending end current or voltage to the receiving end current or voltage.

17. Em waves in free space, dielectric and conductor

Consider the propagation of an electromagnetic wave through a uniform dielectric medium of dielectric constant the dipole moment per unit volume induced in the medium by the wave electric field E . There are no free charges or free currents in the medium. There is also no bound charge density (since the medium is uniform), and no magnetization current density (since the medium is non-magnetic). However, there is a polarization current due to the time-variation of the induced dipole moment per unit volume. According to Eq. (this current is given by since Thus, Maxwell's equations for the propagation of electromagnetic waves through a dielectric medium are the same as Maxwell's equations for the propagation of waves through a vacuum except that where is called the refractive index of the medium in question. Hence, we conclude that electromagnetic waves propagate through a dielectric medium slower than through a vacuum by a factor This conclusion (which was reached long before Maxwell's equations were invented) is the basis of all geometric optics involving refraction. The form of the dielectric constant

Let us investigate an electromagnetic wave propagating through a transparent, isotropic, non-conducting, medium. The electric displacement inside the medium is given by

$$D = \epsilon_0 E + P,$$

where P is the electric polarization. Since electrons are much lighter than ions (or atomic nuclei), we would expect the former to displace further than the latter under the influence of an electric field. Thus, to a first approximation the polarization P is determined by the electron response to the wave. Suppose that the electrons displace a distance s from their rest positions in the presence of the wave. It follows that

include, transmission parameter, transmission function, propagation parameter, propagation coefficient and transmission constant. In plural, it is usually implied that α and β are being referenced separately but collectively as in transmission parameters, propagation parameters, propagation coefficients, transmission constants and secondary coefficients. This last occurs in transmission line theory, the term secondary being used to contrast to the primary line coefficients. The primary coefficients being the physical properties of the line; R, C, L and G, from which the secondary coefficients may be derived using the telegrapher's equation. Note that, at least in the field of transmission lines, the term transmission coefficient has a different meaning despite the similarity of name. Here it is the corollary of reflection coefficient.

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The propagation constant for copper (or any other conductor) lines can be calculated from the primary line coefficients by means of the relationship;

$$\gamma = \sqrt{ZY}$$

Since, by definition,

$$D = \epsilon_0 \epsilon E = \epsilon_0 E + P,$$

it follows that

$$\epsilon(\omega) \equiv n^2(\omega) = 1 + \frac{(N e^2 / \epsilon_0 m)}{\omega_0^2 - \omega^2 - i g \omega \omega_0}.$$

Thus, the index of refraction is frequency dependent. Since ω_0 typically lies in the ultraviolet region of the spectrum (and since $g \ll 1$), it is clear that the denominator $\omega_0^2 - \omega^2 - i g \omega \omega_0 \simeq \omega_0^2 - \omega^2$ is positive in the entire visible spectrum, and is larger at the red end than at the blue end. This implies that blue light is refracted more than red light. This is normal dispersion. Incidentally, an expression, like the above, which specifies the dispersion of waves propagating through some dielectric medium is usually called a dispersion relation.

Let us now suppose that there are N molecules per unit volume with Z electrons per molecule, and that instead of a single oscillation frequency for all electrons, there are f_i electrons per molecule with oscillation frequency ω_i and damping constant g_i . It is easily demonstrated that

$$n^2(\omega) = 1 + \frac{N e^2}{\epsilon_0 m} \sum_i \frac{f_i}{\omega_i^2 - \omega^2 - i g_i \omega \omega_i},$$

where the oscillator strengths f_i satisfy the sum rule,

$$\sum_i f_i = Z.$$

A more exact quantum mechanical treatment of the response of an atom, or molecule, to a low amplitude electromagnetic wave also leads to a dispersion relation of the form (4.18), except that the quantities f_i , ω_i , and g_i can, in principle, be calculated from first principles. In practice, this is too difficult except for the very electromagnetic waves in a conductor

Consider the propagation of an electromagnetic wave through a conducting medium which obeys Ohm's law:

$$P = -N e \dot{s},$$

where N is the number density of electrons. Let us assume that the electrons are bound "quasi-elastically" to their rest positions, so that they seek to return to these positions when displaced from them by a field E . It follows that s satisfies the differential equation of the form

$$m \ddot{s} + f \dot{s} = -e E,$$

where m is the electron mass, $-f \dot{s}$ is the restoring force, and $\dot{}$ denotes a partial derivative with respect to time. The above equation can also be written

$$\ddot{s} + g \omega_0 \dot{s} + \omega_0^2 s = -\frac{e}{m} E,$$

where

$$\omega_0^2 = \frac{f}{m}$$

is the characteristic oscillation frequency of the electrons. In almost all dielectric media this frequency lies in the far ultraviolet region of the electromagnetic spectrum. we have added a phenomenological damping term $g \omega_0 \dot{s}$, in order to take into account the fact that an electron excited by an impulsive electric field does not oscillate for ever. In general, however, electrons in dielectric media can be regarded as high-Q oscillators, which is another way of saying that the dimensionless damping constant g is typically much less than unity. Thus, an electron "rings" for a long time after being excited by an impulse. Let us assume that the electrons oscillate in sympathy with the wave at the wave frequency ω . It follows from

$$s = -\frac{(e/m) E}{\omega_0^2 - \omega^2 - i g \omega \omega_0}.$$

Note that we have neglected the response of the electrons to the magnetic component of the wave. It is easily demonstrated that this is a good approximation provided that the electrons do not oscillate with relativistic velocities (i.e., provided that the amplitude of the wave is sufficiently small). Thus, Eq. (17) yields

$$P = \frac{(N e^2 / m) E}{\omega_0^2 - \omega^2 - i g \omega \omega_0}.$$

$$\vec{E} = E_0 e^{-z/d} e^{i(k_r z - \omega t)}$$

$$d = \frac{2}{\sigma} \sqrt{\frac{\epsilon \epsilon_0}{\mu_0}}$$

where

$$k_r = n \omega / c$$

and

Thus, we conclude that the amplitude of an electromagnetic wave propagating through a conductor decays exponentially on some length-scale, d , which is termed the skin-depth. Note, from Eq that the skin-depth for a poor

conductor is independent of the frequency of the wave. Note, also, that $k_r d \gg 1$ for a poor conductor, indicating that the wave penetrates many wave-lengths into the conductor before decaying away. Consider a "good" conductor for

which $\sigma \gg \epsilon \epsilon_0 \omega$

In this limit, the dispersion relation yields

$$k \simeq \sqrt{i \mu_0 \sigma \omega}$$

Substitution into Eq

$$d = \frac{1}{k_r} = \sqrt{\frac{2}{\mu_0 \sigma \omega}}$$

It can be seen that the skin-depth for a good conductor decreases with increasing

wave frequency. The fact that $k_r d = 1$ indicates that the wave only penetrates a few wave-lengths into the conductor before decaying away.

Now the power per unit volume dissipated via ohmic heating in a conducting medium takes the form

$$P = \vec{j} \cdot \vec{E} = \sigma E^2$$

Consider an electromagnetic wave of the form The mean power dissipated per unit

area in the region $z > 0$ is written

$$\mathbf{j} = \sigma \mathbf{E}.$$

Here, σ is the conductivity of the medium in question. Maxwell's equations for the wave take the form:

$$\nabla \cdot \mathbf{E} = 0,$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \epsilon \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}$$

where ϵ is the dielectric constant of the medium. It follows, from the above equations, that

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E} = -\frac{\partial \nabla \times \mathbf{B}}{\partial t} = -\frac{\partial}{\partial t} \left[\mu_0 \sigma \mathbf{E} + \epsilon \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \right]$$

Looking for a wave-like solution of the form

$$\mathbf{E} = \mathbf{E}_0 e^{i(kz - \omega t)},$$

we obtain the dispersion relation

$$k^2 = \mu_0 \omega (\epsilon \epsilon_0 \omega + i\sigma).$$

$k = \sqrt{\epsilon \mu \omega^2}$
 $\mu_0 \omega (\epsilon \epsilon_0 \omega + i\sigma)$
 $\sigma \ll \epsilon \epsilon_0 \omega$

Consider a "poor" conductor for which relation yields

$$\sigma \ll \epsilon \epsilon_0 \omega$$

In this limit, the dispersion

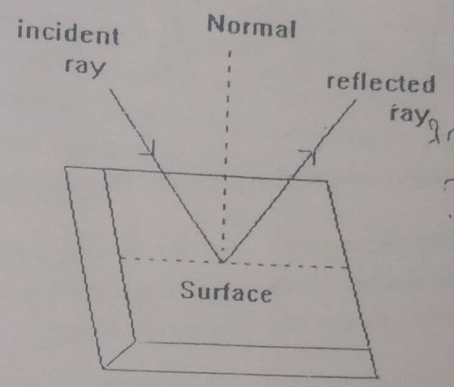
$$k \simeq n \frac{\omega}{c} + i \frac{\sigma}{2} \sqrt{\frac{\mu_0}{\epsilon \epsilon_0}}$$

$$n = \sqrt{\epsilon}$$

where n is the refractive index. Substitution into gives

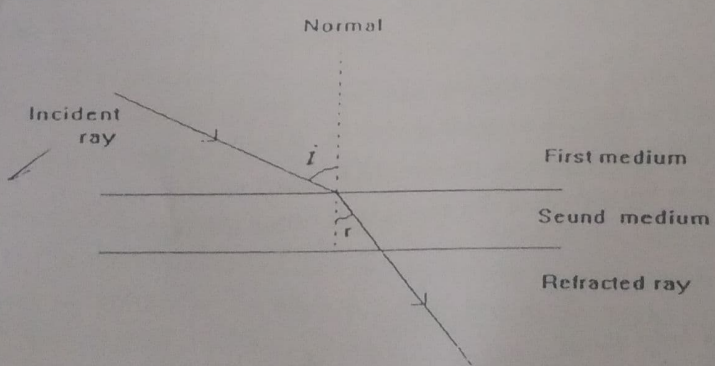
extremely low frequency (ELF) waves (i.e., $f < 100$ Hz). Unfortunately, such waves have very large wave-lengths ($\lambda > 20,000 \text{ km}$), which means that they can only be efficiently generated by extremely large antennas.

4.8. Reflection:



The process in which radiation, meeting the boundary between two media 'bound's back' to stay, in the first medium. Any kind of radiation, wave or stream of particles can be reflected.

4.9. Refraction:



The process in which radiation incident on the boundary between two media passes on into the second medium. Any kind of radiation, wave or stream particles, can be refracted. Refraction in any case depends on the refractive constant.

$$\langle P \rangle = \frac{1}{2} \int_0^\infty \sigma E_0^2 e^{-2z/d} dz = \frac{d\sigma}{4} E_0^2 = \sqrt{\frac{\sigma}{8\mu_0\omega}} E_0^2,$$

for a good conductor. Now, according to Eq. the mean electromagnetic power flux into the region $z > 0$ takes the form

$$\langle u \rangle = \left\langle \frac{\mathbf{E} \times \mathbf{B} \cdot \hat{\mathbf{z}}}{\mu_0} \right\rangle_{z=0} = \frac{1}{2} \frac{E_0^2 k_r}{\mu_0 \omega} = \sqrt{\frac{\sigma}{8\mu_0\omega}} E_0^2.$$

It is clear, from a comparison of the previous two equations, that all of the wave energy which flows into the region $z > 0$ is dissipated via ohmic heating. We thus conclude that the attenuation of an electromagnetic wave propagating through a conductor is a direct consequence of ohmic power losses.

Consider a typical metallic conductor such as copper, whose electrical conductivity at room temperature is about $6 \times 10^7 (\Omega m)^{-1}$. Copper, therefore, acts as a good conductor for all electromagnetic waves of frequency below about 10^{18} Hz. The skin-depth in copper for such waves is thus

$$d = \sqrt{\frac{2}{\mu_0 \sigma \omega}} \approx \frac{6}{\sqrt{\omega(\text{Hz})}} \text{ cm.}$$

It follows that the skin-depth is about 6 cm at 1 Hz, but only about 2 mm at 1 kHz. This gives rise to the so-called skin-effect in copper wires, by which an oscillating electromagnetic signal of increasing frequency, transmitted along such a wire, is confined to an increasingly narrow layer (whose thickness is of order the skin-depth) on the surface of the wire.

The conductivity of sea water is only about $\sigma \approx 5 (\Omega m)^{-1}$. However, this is sufficiently high for sea water to act as a good conductor for all radio frequencies $\omega < 10^9$ Hz.

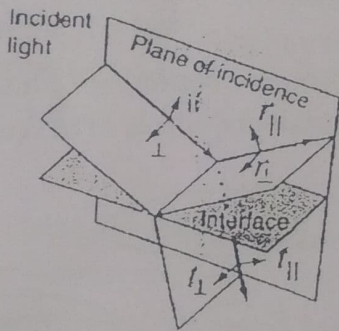
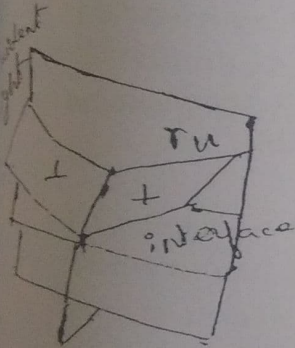
The skin-depth at 1 MHz ($\lambda \sim 300$ m) is still only about 7 cm, whereas that at 1 kHz ($\lambda \sim 300$ m) is obviously quite severe restrictions for radio communication with submarines. Either the submarines have to come quite close to the surface to communicate (which is dangerous), or the communication must be performed with

4.10. Polarization:

Restriction of the vibrations in a transverse wave normally in a transverse wave. The vibration can be any direction in a plane perpendicular to the direction of propagation out way radiation said to be polarized.

4.11. Fresnel's Law

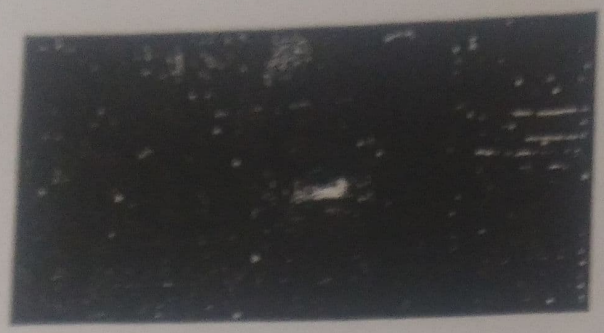
defines the properties of light reflected from a perfectly smooth surface. It is used in physically-based illumination models to describe how light is reflected from micro-facets. Fresnel's equations describe the reflection and transmission of electromagnetic waves at an interface. That is, they give the reflection and transmission coefficients for waves parallel and perpendicular to the plane of incidence. For a dielectric medium where Snell's Law can be used to relate the incident and transmitted angles, Fresnel's Equations can be stated in terms of the angles of incidence and transmission.



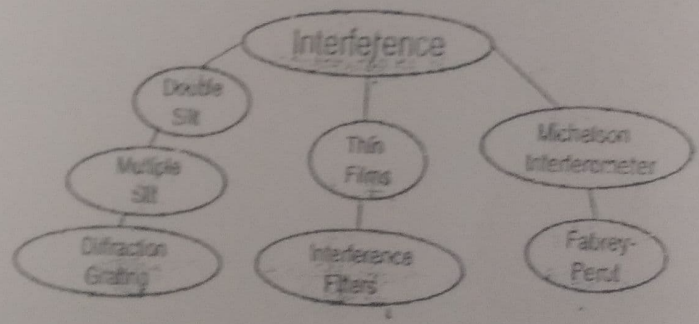
4.12. Interference.

If light is incident onto an obstacle which contains two very small slits a distance d apart, then the wavelets emanating from each slit will interfere behind the obstacle. Waves passing through each slit are diffracted and spread out. At angles where the single slit diffraction pattern produces nonzero intensity, the waves from the two slits can now constructively or destructively interfere.

If we let the light fall onto a screen behind the obstacle, we will observe a pattern of bright and dark stripes on the screen, in the region where with a single slit we only observe a diffraction maximum. This pattern of bright and dark lines is known as an interference fringe pattern.



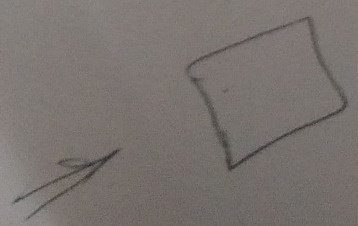
The wave properties of light lead to interference, but certain conditions of coherence must be met for these interference effects to be readily visible.



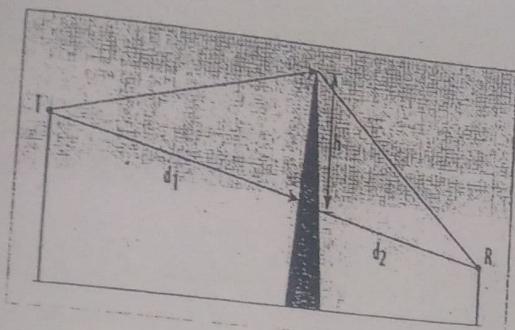
Namathal

4.13. Diffraction

Diffraction refers to various phenomena which occur when a wave encounters an obstacle. In classical physics, the diffraction phenomenon is described as the apparent bending of waves around small obstacles and the spreading out of waves past small openings. Similar effects occur when a light wave travels through a medium with a varying refractive index, or a sound wave travels through one with varying acoustic impedance.



2. The ideal line-of-sight (LOS) propagation of EM waves is disrupted by the presence of large objects that block the signal path.



Radio signals may also undergo diffraction. It is found that when signals encounter an obstacle they tend to travel around them. This can mean that a signal may be received from a transmitter even though it may be "shaded" by a large object between them. This is particularly noticeable on some long wave broadcast transmissions. For example the BBC long wave transmitter on 198 kHz is audible in the Scottish glens where other transmissions could not be heard. As a result the long wave transmissions can be heard in many more places than transmissions on VHF FM.

To understand how this happens it is necessary to look at Huygen's Principle. This states that each point on a spherical wave front can be considered as a source of a secondary wave front. Even though there will be a shadow zone immediately behind the obstacle, the signal will diffract around the obstacle and start to fill the void. It is found that diffraction is more pronounced when the obstacle becomes sharper and more like a "knife edge". For a radio signal a mountain ridge may provide a sufficiently sharp edge. A more rounded hill will not produce such a marked effect. It is also found that low frequency signals diffract more markedly than higher frequency ones. It is for this reason that signals on the long wave band are able to provide coverage even in hilly or mountainous terrain where signals at VHF and higher would not.

4.14. Coherence

Coherence is an ideal property of waves that enables stationary (i.e. temporally and spatially constant) interference. It contains several distinct concepts, which are limit cases that never occur in reality but allow an understanding of the physics of waves, and has become a very important concept in quantum physics. More generally, coherence describes all properties of the correlation between physical quantities of a single wave, or between several waves or wave packets.

Interference is nothing more than the addition, in the mathematical sense, of wave functions. In quantum mechanics, a single wave can interfere with itself, but this is due to its quantum behavior and is still an addition of two waves (see Young's slits

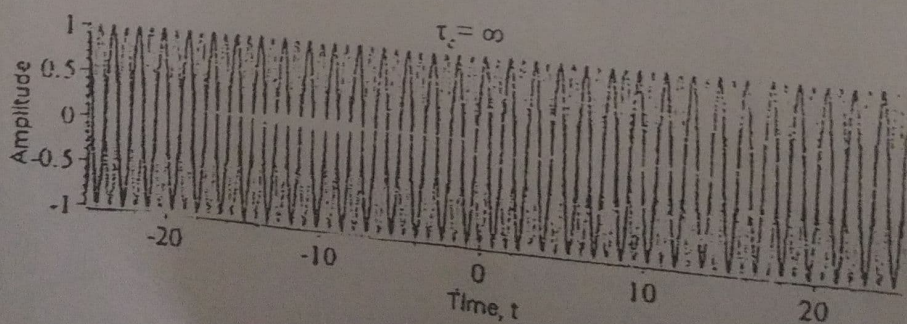
experiment). This implies that constructive or destructive interferences are limit cases, and that waves can always interfere, even if the result of the addition is complicated or not remarkable.

When interfering, two waves can add together to create a wave of greater amplitude than either one (constructive interference) or subtract from each other to create a wave of lesser amplitude than either one (destructive interference), depending on their relative phase. Two waves are said to be coherent if they have a constant relative phase. The degree of coherence is measured by the interference, a measure of how perfectly the waves can cancel due to destructive interference.

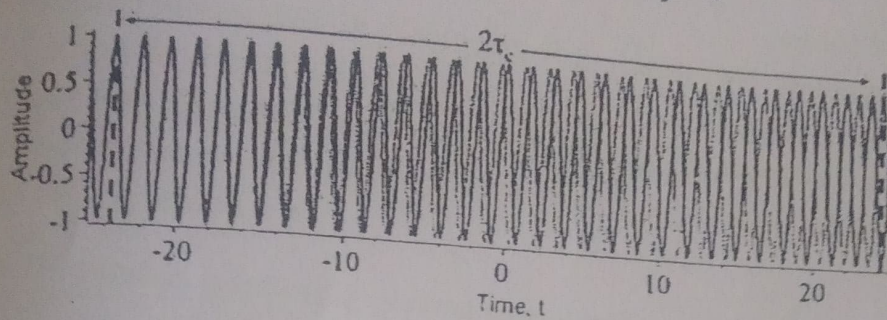
Spatial coherence describes the correlation between waves at different points in space. Temporal coherence describes the correlation or predictable relationship between waves observed at different moments in time. Both are observed in the experiment and Young's interference experiment. Once the fringes are obtained in the Michelson-Morley experiment, when one of the mirrors is moved away gradually, the time for the beam to travel increases and the fringes become dull and finally are lost, showing temporal coherence. Similarly, if in Young's double slit experiment the space between the two slits is increased, the coherence dies gradually and finally the fringes disappear, showing spatial coherence.

Temporal coherence

The coherence of two waves follows from how well correlated the waves are as quantified by the cross-correlation function. The cross-correlation quantifies the ability to predict the value of the second wave by knowing the value of the first. As an example, consider two waves perfectly correlated for all times. At any time, if the first wave changes, the second will change in the same way. If combined they can exhibit complete constructive interference/superposition at all times, then it follows that they are perfectly coherent. As will be discussed below, the second wave need not be a separate entity. It could be the first wave at a different time or position. In this case, the measure of correlation is the autocorrelation function (sometimes called self-coherence). Degree of correlation involves correlation functions.



The amplitude of a single frequency wave as a function of time t (red) and a copy of the same wave delayed by τ (green). The coherence time of the wave is infinite since it is perfectly correlated with itself for all delays τ .



: The amplitude of a wave whose phase drifts significantly in time τ_c as a function of time t (red) and a copy of the same wave delayed by $2\tau_c$ (green). At any particular time t the wave can interfere perfectly with its delayed copy. But, since half the time the red and green waves are in phase and half the time out of phase, when averaged over t any interference disappears at this delay.

Temporal coherence is the measure of the average correlation between the value of a wave and itself delayed by τ , at any pair of times. Temporal coherence tells us how monochromatic a source is. In other words, it characterizes how well a wave can interfere with itself at a different time. The delay over which the phase or amplitude wanders by a significant amount (and hence the correlation decreases by significant amount) is defined as the coherence τ_c . At $\tau=0$ the degree of coherence is perfect whereas it drops significantly by delay τ_c . The coherence length L_c is defined as the distance the wave travels in time τ_c .

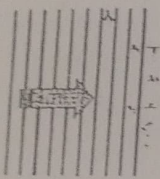
Spatial coherence

In some systems, such as water waves or optics, wave-like states can extend over one or two dimensions. Spatial coherence describes the ability for two points in space, x_1 and x_2 , in the extent of a wave to interfere, when averaged over time. More precisely, the spatial coherence is the cross-correlation between two points in a wave for all times. If a wave has only 1 value of amplitude over an infinite length, it is perfectly spatially coherent. The range of separation between the two points over which there is significant interference is called the coherence area, A_c . This is the relevant type of coherence for the Young's double-slit interferometer. It is also used in optical imaging systems and particularly in various types of astronomy

telescopes. Sometimes people also use "spatial coherence" to refer to the visibility when a wave-like state is combined with a spatially shifted copy of itself.

Examples of spatial coherence

Spatial coherence



: A plane wave with an infinite coherence length.

4.15. The Poynting Vector

The Poynting vector usually written as S is the direction in which energy travels in an EM wave, we will not go into the vector calculus, but it is given by taking the cross product of the vector field of E and the complex conjugate of the vector field H .

$$S = E \times H^*$$

This represents a power flow along the z axis. The average in watts per square metre is given by: $S_{av} = \frac{1}{2} E_0 H_0 z$ Watts/m²

The phase velocity is the rate along the z axis that a point of constant phase moves,

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

which is the speed of light and is approximately 3×10^8 m/s in free space and the wavelength is the velocity divided by the frequency:

$$\lambda = \frac{v}{f}$$

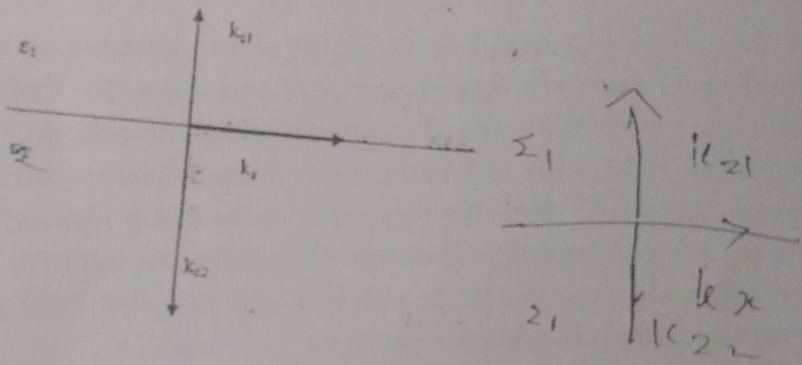
The Exponential Notation

It is also possible for us to express E and H in exponential notation:

$$E(z, t) = E_0 \mathbf{x} \cdot \text{Re} \{ e^{j(\omega t - kz)} \}$$

Where $\text{Re}\{\}$ means take the real part. This comes from the equivalence $e^{jx} = \cos(x) + j \sin(x)$. All this is very useful as by using this notation differentiation is easy because the differential of $e^{jx} = -je^{jx}$. It basically makes the maths of the plane wave equation easier because of the relative ease of taking the differential. From the point of view in question, i.e.

4.16. Dispersion relation in plasma skin depth



Coordinate system for 2 material interface

The electric field of a propagating electromagnetic wave can be expressed

$$E = E_0 \exp[i(k_x x + k_z z - \omega t)] \quad E = E_0 \exp [i (k_x x + k_z z - \omega t)]$$

where k is the wave number and ω is the frequency of the wave. By solving Maxwell's equations for the electromagnetic wave at an interface between two materials with relative dielectric functions ϵ_1 and ϵ_2 (see figure 3) with the appropriate continuity relation the boundary conditions are

$$\frac{k_{z1}}{\epsilon_1} + \frac{k_{z2}}{\epsilon_2} = 0 \quad \frac{k_{x1}}{\epsilon_1} + \frac{k_{x2}}{\epsilon_2} = 0$$

and

$$k_x^2 + k_{zi}^2 = \epsilon_i \left(\frac{\omega}{c} \right)^2 \quad i = 1, 2$$

where c is the speed of light in a vacuum, and k_x is same for both media interface for a surface wave. Solving these two equations, the dispersion relation for a wave propagating on the surface is

$$k_x = \frac{\omega}{c} \left(\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \right)^{1/2}$$

Dispersion curve for surface Plasmon's. At low k , the surface Plasmon curve (red) approaches the photon curve (blue) In the free electron model of an electron gas,

which neglects attenuation, the metallic dielectric function is

$$\epsilon(\omega) = 1 - \frac{\omega_P^2}{\omega^2},$$

where the bulk plasma frequency in SI units is

$$\omega_P = \sqrt{\frac{ne^2}{\epsilon_0 m^*}}$$

where n is the electron density, e is the charge of the electron, m^* is the effective mass of the electron and ϵ_0 is the permittivity of free-space. The dispersion relation is plotted in Figure 4. At low k , the SP behaves like a photon, but as k increases, the dispersion relation bends over and reaches an asymptotic limit corresponding to the surface plasma frequency. Since the dispersion curve lies to the right of the light line, $\omega = kc$, the SP has a shorter wavelength than free-space radiation such that the out-of-plane component of the SP is purely imaginary and exhibits evanescent decay. The surface plasma frequency is given by

$$\omega_{SP} = \omega_P / \sqrt{1 + \epsilon_2}.$$

In the case of air, this result simplifies to

$$\omega_{SP} = \omega_P / \sqrt{2}.$$

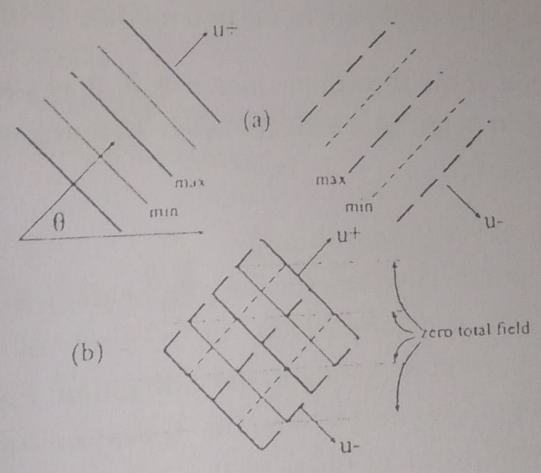
If we assume that ϵ_2 is real and $\epsilon_2 > 0$, then it must be true that $\epsilon_1 < 0$, a condition which is satisfied in metals. Electromagnetic waves passing through a metal experience damping due to Ohmic losses and electron-core interactions. These effects show up in as an imaginary component of the dielectric function. The dielectric function of a metal is expressed $\epsilon_1 = \epsilon_1' + i \epsilon_1''$ where ϵ_1' and ϵ_1'' are the real and imaginary parts of the dielectric function, respectively. Generally $|\epsilon_1'| \gg \epsilon_1''$ so the wavenumber can be expressed in terms of its real and imaginary components. The wave vector gives us insight into physically meaningful properties of the electromagnetic wave such as its spatial extent and coupling requirements for wave vector matching.

4.17. Wave Guides

- > Rectangular Waveguides
- > TEM, TE and TM waves
- > Cutoff Frequency
- > Wave Propagation

Now consider a pair of identical TEM waves, labeled as u^+ and u^- in Figure (a). The u^+ wave is propagating at an angle $+\theta$ to the z axis, while the u^- wave propagates at an angle $-\theta$. These waves are combined in Figure (b). Notice that horizontal lines can be drawn on the superposed waves that correspond to zero field. Along these lines the u^+ wave is always 180° out of phase with the u^- wave. Since we know $E = 0$ on a perfect conductor, we can replace the horizontal lines of zero-field with perfect conducting walls. Now, u^+ and u^- are reflected off the walls as they propagate along the guide.

The distance separating adjacent zero-field lines in Figure (b), or separating the conducting walls in Figure (a), is given as the dimension a in Figure (b). The distance a is determined by the angle θ and by the distance between wavefront peaks, or the wavelength λ . For a given wave velocity u , the frequency is $f = u/\lambda$. If we fix the wall separation at a , and change the frequency, we must then also change the angle θ if we are to maintain a propagating wave. Figure (b) shows wave fronts for the u^+ wave. The edge of a $+E_0$ wave front (point A) will line up with the edge of a $-E_0$ front (point B), and the two fronts must be $\lambda/2$ apart for the $m = 1$ mode.

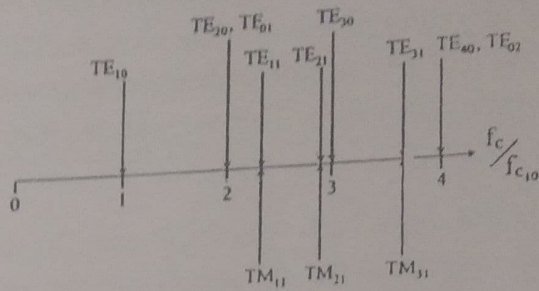


A constant phase point is on the wall from A to D. Calling this phase velocity u_p , and given the distance along the wall is $l_{AD} = \frac{m\lambda/2}{\cos\theta}$

$$l_{AD} = \frac{m\lambda/2}{\cos\theta}$$

Then the time t_{AD} to travel from A to D is

$$t_{AD} = \frac{l_{AD}}{u_p} = \frac{m\lambda/2}{\cos\theta u_p}$$



$$f_{c_{mn}} = \frac{c}{2\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad \longrightarrow \quad f_{c_{mn}} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

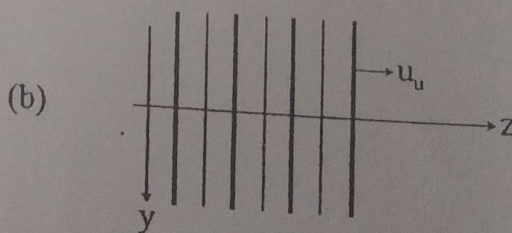
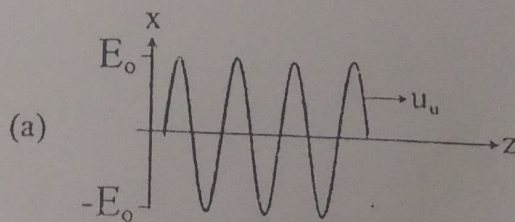
Let us take a look at the field pattern for two modes, TE10 and TE20

In both cases, E only varies in the x direction; since n = 0, it is constant in the y direction. For TE10, the electric field has a half sine wave pattern, while for TE20 a full sine wave pattern is observed.

4.18. Rectangular Waveguide - Wave Propagation

We can achieve a qualitative understanding of wave propagation in waveguide by considering the wave to be a superposition of a pair of TEM waves.

Let us consider a TEM wave propagating in the z direction. Figure shows the wave fronts; bold lines indicating constant phase at the maximum value of the field (+E₀), and lighter lines indicating constant phase at the minimum value (-E₀). The waves propagate at a velocity u_u, where the u subscript indicates media unbounded by guide walls. In air, u_u = c.



where

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t}$$

and

$$\mathbf{B} = \nabla \times \mathbf{A}.$$

If the Lorenz gauge condition is assumed

$$\frac{1}{c^2} \frac{\partial \varphi}{\partial t} + \nabla \cdot \mathbf{A} = 0$$

then the nonhomogeneous wave equations become

$$\nabla^2 \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}$$

4.20. Retarded potentials

the same form as the inhomogeneous wave equation (2.103), so we can immediately write the solutions to these equations as

$$\phi(\mathbf{r}, t) = \frac{1}{4\pi \epsilon_0} \int \frac{[\rho(\mathbf{r}')]_{\text{ret}}}{|\mathbf{r} - \mathbf{r}'|} dV',$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{[\mathbf{j}(\mathbf{r}')]_{\text{ret}}}{|\mathbf{r} - \mathbf{r}'|} dV'.$$

Moreover, we can be sure that these solutions are unique, subject to the reasonable proviso that infinity is an absorber of radiation but not an emitter. This is a crucially important point. Whenever the above solutions are presented in physics textbooks there is a tacit assumption that they are unique. After all, if they were not unique why should we choose to study them instead of one of the other possible solutions? The uniqueness of the above solutions has a physical interpretation. It is clear from Eqs. (2.141) that in the absence of any charges and currents there are no electromagnetic fields. In other words, if we observe an electromagnetic field we can be certain that if we were to trace it backward in time we would eventually discover that it was emitted by a charge or a current. In proving that the solutions of Maxwell's equations are unique, and then finding a solution in which all waves are

Since the times t_{AD} and t_{AC} must be equal, we have

4.19. Inhomogeneous Electromagnetic Wave Equation

Localized time-varying charge and current densities can act as sources of electromagnetic waves in a vacuum. Maxwell's equations can be written in the form of an inhomogeneous electromagnetic wave equation (or often "nonhomogeneous electromagnetic wave equation") with sources. The addition of sources to the wave equations makes the partial differential equation inhomogeneous.

The relativistic Maxwell's equations can be written in covariant form as

$$\square A^\mu \stackrel{\text{def}}{=} \partial_\beta \partial^\beta A^\mu \stackrel{\text{def}}{=} A^{\mu;\beta}{}_\beta = -\mu_0 J^\mu (SI)$$

$$\square A^\mu \stackrel{\text{def}}{=} \partial_\beta \partial^\beta A^\mu \stackrel{\text{def}}{=} A^{\mu;\beta}{}_\beta = -\frac{4\pi}{c} J^\mu (cgs)$$

where J is the four-current

$$J^\mu = (c\rho, \mathbf{J}),$$

$$\frac{\partial}{\partial x^a} \stackrel{\text{def}}{=} \partial_a \stackrel{\text{def}}{=} \cdot_a \stackrel{\text{def}}{=} (\partial/\partial ct, \nabla)$$

is the 4-gradient and the electromagnetic four-potential is

$$A^\mu = (\varphi, \mathbf{A}c) (SI)$$

$$A^\mu = (\varphi, \mathbf{A}) (cgs)$$

with the Lorenz gauge condition

$$\partial_\mu A^\mu = 0$$

Here

$$\square = \partial_\beta \partial^\beta = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \text{ is the d'Alembert operator.}$$

Maxwell's equations in a vacuum with charge ρ and current \mathbf{J} sources can be written in terms of the vector and scalar potentials as

$$\nabla^2 \varphi + \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla \left(\frac{1}{c^2} \frac{\partial \varphi}{\partial t} + \nabla \cdot \mathbf{A} \right) = -\mu_0 \mathbf{J}$$

emitted by sources, we have effectively ruled out the possibility that the vacuum can be "unstable" to the production of electromagnetic waves without the need for any sources.

$$\Phi^\mu = \frac{1}{4\pi \epsilon_0 c} \int \frac{[J^\mu]}{r} dV.$$

Here, the components of the 4-potential are evaluated at some event P in space-time, r is the distance of the volume element dV from P , and the square brackets indicate that the 4-current is to be evaluated at the retarded time; i.e., at a time before P . We shall now demonstrate that all observers obtain the same value of dV/r for each elementary contribution to the integral.

Suppose that S and S' are two inertial frames in the standard configuration. Let unprimed and primed symbols denote corresponding quantities in S and S' , respectively. Let us assign coordinates $(0,0,0,0)$ to P and (x,y,z,t) to the retarded event Q for which r and dV are evaluated. Using the standard Lorentz transformation (2.19), the fact that the interval between events P and Q is zero, and the fact that both t and t' are negative, we obtain

$$r' = -ct' = -c\gamma \left(t - \frac{vx}{c^2} \right),$$

where v is the relative velocity between frames S' and S , γ is the Lorentz factor,

and $r = \sqrt{x^2 + y^2 + z^2}$, etc. It follows that

$$r' = \gamma \left(-\frac{ct}{r} + \frac{vx}{cr} \right) = \gamma \left(1 + \frac{v}{c} \cos \theta \right),$$

where θ is the angle (in 3-space) subtended between the line PQ and the x -axis.

We now know the transformation for r . What about the transformation for dV ? We might be tempted to set

$dV' = \gamma dV$, according to the usual length contraction rule.

However, this is wrong. The contraction by a factor γ only applies if the whole of the volume is measured at the same time, which is not the case in the present

problem. Now, the dimensions of dV along the y and z axes are the same in both S and S' , according to Eqs. (2.19). For the x -dimension these equations

give $d x' = \gamma(dx - v dt)$. The extremities of dx are measured at times differing by dt , where $\underline{5}$

$$dt = -\frac{dr}{c} = -\frac{dx}{c} \cos \theta.$$

Thus,

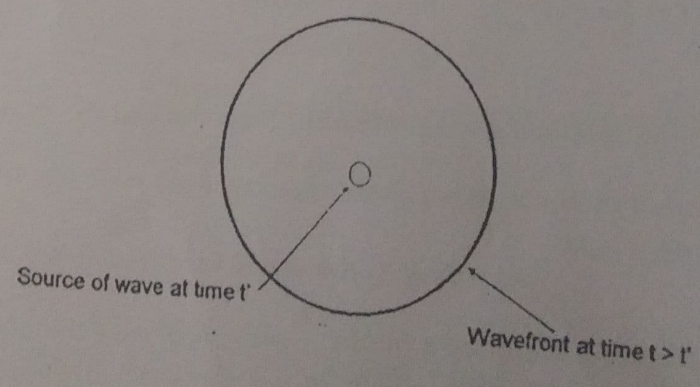
$$dx' = \left(1 + \frac{v}{c} \cos \theta\right) \gamma dx,$$

giving

$$dV' = \left(1 + \frac{v}{c} \cos \theta\right) \gamma dV.$$

It follows from Eqs. (2.144) and (2.147) that $dV'/r' = dV/r$. This result will clearly remain valid even when S and S' are not in the standard configuration

Thus, dV/r is an invariant and, therefore, $[J^\mu]dV/r$ is a contra variant 4-vector. For linear transformations, such as a general Lorentz transformation, the result of adding 4-tensors evaluated at different 4-points is itself a 4-tensor. It follows that the right-hand side of Eq. (2.142) is a contra variant 4-vector. Thus, this 4-vector equation can be properly regarded as the solution to the 4-vector wave equation (2.96).



Retarded spherical wave. The source of the wave occurs at time t' . The wave front moves away from the source as time increases for $t > t'$. For advanced solutions, the wave front moves backwards in time from the source $t < t'$.

In the case that there are no boundaries surrounding the sources, the solutions (cgs units) of the nonhomogeneous wave equations are

$$\varphi(\mathbf{r}, t) = \int \frac{\delta\left(t' + \frac{|\mathbf{r} - \mathbf{r}'|}{c} - t\right)}{|\mathbf{r} - \mathbf{r}'|} \rho(\mathbf{r}', t') d^3r' dt'$$

and

$$\mathbf{A}(\mathbf{r}, t) = \int \frac{\delta\left(t' + \frac{|\mathbf{r} - \mathbf{r}'|}{c} - t\right)}{|\mathbf{r} - \mathbf{r}'|} \frac{\mathbf{J}(\mathbf{r}', t')}{c} d^3r' dt'$$

where

$$\delta\left(t' + \frac{|\mathbf{r} - \mathbf{r}'|}{c} - t\right)$$

is a Dirac delta function.

For SI units

$$\rho \rightarrow \frac{\rho}{4\pi\epsilon_0}$$

$$\mathbf{J} \rightarrow \frac{\mu_0}{4\pi} \mathbf{J}$$

For Lorentz-Heaviside units,

$$\rho \rightarrow \frac{\rho}{4\pi}$$

$$\mathbf{J} \rightarrow \frac{1}{4\pi} \mathbf{J}$$

These solutions are known as the retarded Lorenz gauge potentials. They represent a superposition of spherical light waves traveling outward from the sources of the waves, from the present into the future.

There are also advanced solutions (cgs units)

$$\varphi(\mathbf{r}, t) = \int \frac{\delta\left(t' - \frac{|\mathbf{r} - \mathbf{r}'|}{c} - t\right)}{|\mathbf{r} - \mathbf{r}'|} \rho(\mathbf{r}', t') d^3r' dt'$$

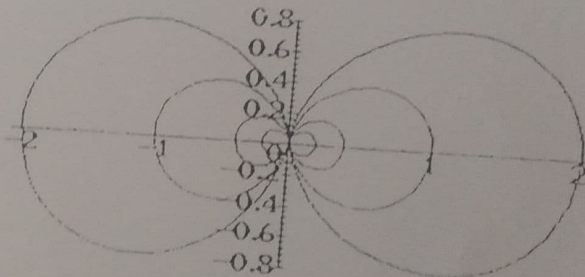
and

$$A(\mathbf{r}, t) = \int \frac{\delta\left(t' - \frac{|\mathbf{r} - \mathbf{r}'|}{c} - t\right) \mathbf{J}(\mathbf{r}', t')}{|\mathbf{r} - \mathbf{r}'| c} d^3r' dt'$$

These represent a superposition of spherical waves travelling from the future into the present.

4.2.1. Radiation from an Oscillating Electric Dipole

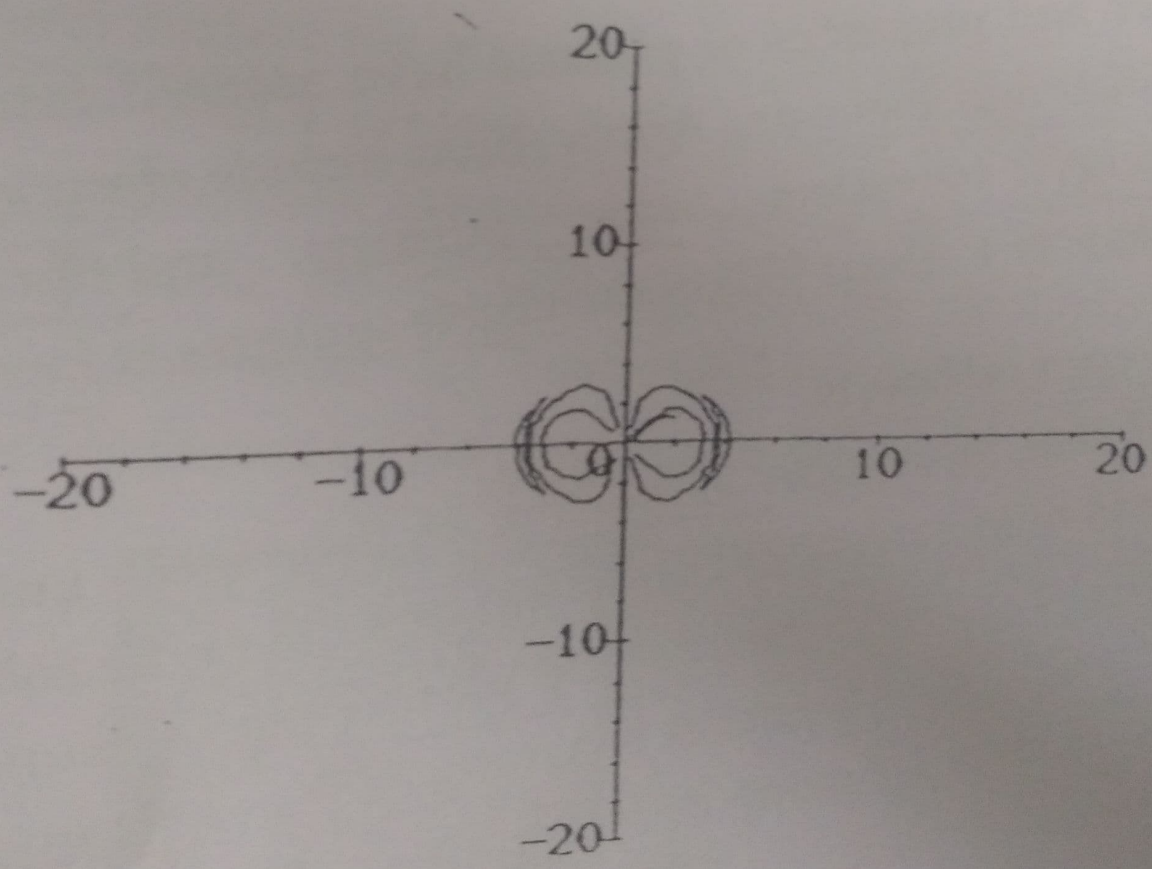
The basic mechanism of electromagnetic radiation is discussed in Note 10. For a charge to radiate electromagnetic waves in free space, it must be either accelerated or decelerated. Charges undergoing oscillatory motion are continuously accelerated and decelerated and if the velocity of charges is nonrelativistic, electromagnetic waves are radiated at the oscillation frequency. Radiation from antennas can be analyzed by superposing elementary radiation fields emitted by individual electrons as shown in Note 10.



Electric field lines of static dipole.

Animation shows the electric field lines due to an electric dipole oscillating vertically at the origin. Near the dipole, the field lines are essentially those of a static dipole leaving a positive charge and ending up at a negative charge. However, at a distance of the order of half wavelength ($\lambda = 2\pi$ is assumed here) or greater, the field lines are completely detached from the dipole. This detachment characterizes radiation fields which propagate freely (without being attached to charges) in free space at the speed c .

Rectangular wave guide
Poynting vector



UNIT V

