Operations Research.

UNIT : II : Assignment Problems

1. **Definition:** Assignment Problem.

The assignment problem is a special case of the transportation problem in which the objective is to assign a number of resources to the equal numbers of activities at a minimum cost.

2. Mathematical formulation of the assignment problem:

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}.$$

Subject to the constraints.

$$\sum_{i=1}^{n} x_{ij} = 1$$

and $$\sum_{j=1}^{n} x_{ij} = 1$$

$x_{ij} = 0$ (or) 1 , for all $i = 1, 2 \ldots n$ and $j = 1, 2 \ldots n$.

3. Given below is an assignment problem. Write it as a transportation problem:

	A_1	A_2	A_3
R_1	1	2	3
R_2	4	5	1
R_3	2	1	4

Answer:

Let x_{ij} denote the assignment of R_i $(i = 1, 2, 3)$ to A_j $(j = 1, 2, 3)$ such that

$$x_{ij} = \begin{cases} 1, & \text{if } R \text{ is assigned to } A_{ij} \\ 0, & \text{Otherwise} \end{cases}$$

Then, the transportation problem is,

Minimize $Z = 1.x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + 5x_{22} + 1.x_{23} + 2.x_{31} + 1.x_{32} + 4.x_{33}$.

Subject to the constraints,

$$\left.\begin{aligned} x_{11}+x_{12}+x_{13} &= 1 \\ x_{21}+x_{22}+x_{23} &= 1 \\ x_{31}+x_{32}+x_{33} &= 1 \end{aligned}\right\} \qquad \left.\begin{aligned} x_{11}+x_{21}+x_{31} &= 1 \\ x_{12}+x_{22}+x_{32} &= 1 \\ x_{13}+x_{23}+x_{33} &= 1 \end{aligned}\right\}$$

$x_{ij} = 0$ (or) 1, for $i = 1,2,3$ and $j = 1,2,3$.

4. The assignment problem is a variation of transportation problem with two characteristics:

(i). The cost matrix is a square matrix

(ii). The optimum solution for the problem would always be such that there would be only one assignment in a given row or column of the cost matrix.

5. What is an unbalanced assignment problem!

If the cost matrix of an assignment problem is not a square matrix, then the problem is called an unbalanced assignment problem.

6. How to solve an unbalanced assignment problem:

If the given problem is an unbalanced once, then add number of rows or columns with all the entries are zero, so as to form a square matrix. Then the usual assignment algorithm can be applied to this resulting balanced inorder to obtain optimum assignment.

7. How would you deal with the assignment problem where the objective function is to maximized?

When an assignment problem involves maximizing objective function such a problem may be solved by converting it into a minimization problem in the following way.

(i) Put a negative sign before each payoff element in the assignment table so as to convert the profit values in the cost values. (or).

(ii) Locate the longest payoff element ~~also~~ in the assignment table and then subtract all the elements of the table from this largest element.

The transformed assignment problem so as to obtained can be solved by using Hungarian method.

Note:

The method to solve an assignment problem is Hungarian assignment method.

Hungarian Assignment Method: (Reduce matrix Method)

An effective method for solving an assignment problem is the Hungarian Assignment method.

Step: 1

Determine the cost table from the given problem.

(i) If the number of sources is equal to number of destinations go to step 3.

(ii). If the no. of sources is not equal to the number of destinations go to step 2.

Step: 2

Add a dummy source (or) dummy destination so that the cost table becomes a square matrix. The cost entries of dummy source/destinations are always zero.

Step : 3

Locate the smallest element in each row of the given cost matrix and then subtract the same from each element of that row.

Step : 4

In the reduced matrix, obtained in step 3, locate the smallest element of each column and then subtract the same from each element of that column. Each column and row now have atleast one zero.

Step : 5

In the modified matrix obtained in step 4, search for an optimal assignment as follows:

a) Examine the rows successively until a row with a single zero is found. Enrectangle this zero (□) and cross off (x) all other zeros in its column.

b) Repeat the procedure for each column of the reduced matrix.

c) If a row and / or column has two or more zeroes and one cannot be chosen by Inspection, then assign arbitrarily any one of these zeroes and cross off all other zeroes of that row / column.

d) Repeat (a) through (c) above successively until the chain of assigning (□) or cross (x) ends.

Step : 6 :

If the number of assignment (□) is equal to n (the order of the cost matrix) an optimum solution is reached.

If the number of assignments is less than n go to next step.

Scanned by TapScanner

Step: 7

Draw the minimum number of horizontal and/or vertical lines to cover all the zeroes of the reduced matrix. This can be conveniently done by using a simple procedure.

a) ~~Matrix~~ Mark (✓) rows that donot have any assigned zero.

b) Mark (✓) columns that have zeroes in the marked rows.

c) Mark (✓) rows that have assigned zeroes in the marked columns..

d) Repeat (b) and (c) above until the chain of marking is completed.

e) Draw lines through all the unmarked rows and marked columns. This give us the desired minimum number of lines.

Step: 8

Develop the new revised cost matrix as follows:

a) Find the smallest element of the reduced matrix not covered by any of the lines.

b) Subtract this element from all the uncovered elements and add the same to all the elements lying at the intersection of any two lines.

Step: 9

Go to step 6 and repeat the procedure until an optimum solution is attained.

Problems :

Solve the following Assignment Problems!

1.

	A	B	C	D
1	10	25	15	20
2	15	30	5	15
3	35	20	12	24
4	17	25	24	20

<u>Solution:</u>

No. of rows = No. of columns.

It is a balanced Assignment Problem.

<u>Reduced Row matrix:</u>

0	15	5	10
10	25	0	10
23	8	0	12
0	8	7	3

<u>Reduced column matrix</u>

$\boxed{0}$	7	5	7
10	17	$\boxed{0}$	7
23	$\boxed{0}$	~~0~~	9
~~0~~	~~0~~	7	$\boxed{0}$

No. of Assignment = Order of the matrix.

∴ The optimum solution is

$1 \rightarrow A$, $2 \rightarrow C$, $3 \rightarrow B$, $4 \rightarrow D$

Assignment Value = 10 + 5 + 20 + 20

= Rs. 55

2.

	1	2	3	4
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

Solution:

No. of rows = No of columns

It is a balanced Assignment Problem.

Reduced Row matrix:

$$\begin{matrix} 0 & 2 & 9 & 1 \\ 0 & 5 & 2 & 3 \\ 1 & 3 & 2 & 0 \\ 0 & 7 & 3 & 1 \end{matrix}$$

Reduced column matrix

$$\begin{matrix} \cancel{0} & \boxed{0} & 7 & 1 \\ \cancel{0} & 3 & \boxed{0} & 3 \\ 1 & 1 & \cancel{0} & \boxed{0} \\ \boxed{0} & 5 & 1 & 1 \end{matrix}$$

No. of Assignment = Order of the matrix

∴ The optimum solution is

$A \rightarrow 2$, $B \rightarrow 3$, $C \rightarrow 4$, $D \rightarrow 1$.

The Assignment value = 12 + 7 + 11 + 8

= Rs. 38.

3.

	A	B	C
I	8	7	6
II	5	7	8
III	6	8	7.

Scanned by TapScanner

Solution:

No. of rows = No. of columns.

It is a balanced Assignment problem.

Reduced Row matrix

$$\begin{matrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 2 & 1 \end{matrix}$$

Reduced Column matrix

$$\begin{matrix} 2 & \boxed{0} & \cancel{0} \\ \boxed{0} & 1 & 3 & \checkmark \\ \cancel{0} & 1 & 1 & \checkmark \\ \checkmark & & & \end{matrix}$$

Iteration: 1

$$\begin{matrix} 3 & \boxed{0} & \cancel{0} \\ \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & \boxed{0} \end{matrix}$$

No. of Assignment = order of the matrix

∴ The optimum solution is

$I \rightarrow B$, $II \rightarrow A$, $III \rightarrow C$.

Assignment value = 4 + 5 + 7 = Rs. 19.

4.

	1	2	3	4
A	2	10	9	7
B	15	4	14	8
C	13	14	16	11
D	4	15	13	9

Solution:

No. of rows = No. of columns.

It is a balanced Assignment Problem.

Reduced Row matrix:

0	8	7	5
11	0	10	4
2	3	5	0
0	11	9	5

Reduced column matrix:

$\boxed{0}$	8	2	5	✓
11	$\boxed{0}$	5	4	
2	3	$\boxed{0}$	$\cancel{0}$	
$\cancel{0}$	11	4	5	✓
✓				

Iteration: 1

$\cancel{0}$	6	$\boxed{0}$	3
13	$\boxed{0}$	5	4
4	3	$\cancel{0}$	$\boxed{0}$
$\boxed{0}$	9	2	3

No. of Assignment = Order of matrix

∴ The optimum solution is

$A \rightarrow 3, \quad B \rightarrow 2, \quad C \rightarrow 4, \quad D \rightarrow 1$

Assignment Value $= 9 + 4 + 11 + 4 =$ Rs. 28.

Scanned by TapScanner

Homework!

1)

$$\begin{array}{c|ccc} & I & II & III \\ A & 10 & 8 & 12 \\ B & 18 & 6 & 14 \\ C & 6 & 4 & 2 \end{array}$$

2)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ A & 2 & 10 & 9 & 7 \\ B & 15 & 4 & 14 & 8 \\ C & 13 & 14 & 16 & 11 \\ D & 4 & 15 & 13 & 9 \end{array}$$

3)

$$\begin{array}{c|ccccc} & I & II & III & IV & V \\ 1 & 11 & 17 & 8 & 16 & 20 \\ 2 & 9 & 7 & 12 & 6 & 15 \\ 3 & 13 & 16 & 15 & 12 & 16 \\ 4 & 21 & 24 & 17 & 28 & 26 \\ 5 & 14 & 10 & 12 & 11 & 15 \end{array}$$

4)

$$\begin{array}{c|ccccc} & I & II & III & IV & V \\ 1 & 3 & 8 & 2 & 10 & 3 \\ 2 & 8 & 7 & 2 & 9 & 7 \\ 3 & 3 & 6 & 4 & 2 & 5 \\ 4 & 8 & 4 & 2 & 3 & 5 \\ 5 & 9 & 10 & 6 & 9 & 10 \end{array}$$

5.

$$\begin{array}{c|cccc|}
 & I & II & III & IV \\
\hline
A & M & 5 & 2 & 0 \\
B & 4 & 7 & 5 & 6 \\
C & 5 & 8 & 4 & 3 \\
D & 3 & 6 & 6 & 2 \\
\hline
\end{array}$$

Solution:

No. of rows = No. of Columns.

It is a balanced Assignment problem.

Reduced Row Matrix:

$$\begin{array}{cccc}
M & 5 & 2 & 0 \\
0 & 3 & 1 & 2 \\
2 & 5 & 1 & 0 \\
1 & 4 & 4 & 0
\end{array}$$

Reduced column matrix:

$$\begin{array}{cccc}
M & 2 & 1 & \boxed{0} \\
\boxed{0} & \cancel{0} & \cancel{0} & 2 \\
2 & 2 & \boxed{0} & \cancel{0} \\
1 & 1 & 3 & \cancel{0}
\end{array}$$

Iteration: 1

$$\begin{array}{cccc}
M & 1 & \cancel{0} & \boxed{0} \\
\boxed{0} & \cancel{0} & \cancel{0} & 3 \\
2 & 2 & \boxed{0} & 1 \\
\cancel{0} & \boxed{0} & 2 & \cancel{0}
\end{array}$$

No. of Assignment = Order of the matrix

∴ The optimum solution is.

A → IV , B → I , C → III , D → II .

Assignment value = 0+4+4+6 = Rs. 14 .

6.

	A'	B'	C'	D'	E'
A	10	5	9	18	11
B	13	19	6	12	14
C	3	2	4	4	5
D	18	9	12	17	15
E	11	6	14	19	10

Solution:

No. of rows = No. of columns.

It is a balanced Assignment problem.

Reduced Row matrix:

5	0	4	13	6
7	13	0	6	8
1	0	2	2	3
9	0	3	8	6
5	0	8	13	4.

Reduced column matrix:

4	$\boxed{0}$	4	11	3	✓
6	13	$\boxed{0}$	4	5	
$\boxed{0}$	$\cancel{0}$	2	$\cancel{0}$	$\cancel{0}$	
8	$\cancel{0}$	3	6	3	✓
4	$\cancel{0}$	8	11	1	✓

✓

Iteration: 1

3	$\boxed{0}$	3	10	2	✓
6	14	$\boxed{0}$	4	5	
$\boxed{0}$		2	$\cancel{0}$	$\cancel{0}$	
7	$\cancel{0}$	2	5	2	✓
3	$\cancel{0}$	7	10	$\boxed{0}$	
	✓				

Iteration: 2

1	$\boxed{0}$	1	8	$\cancel{0}$	✓
6	16	$\boxed{0}$	4	5	✓
$\boxed{0}$	3	2	$\cancel{0}$	$\cancel{0}$	
5	$\cancel{0}$	$\cancel{0}$	3	$\cancel{0}$	✓
3	2	7	10	$\boxed{0}$	✓
	✓	✓		✓	

Iteration: 3

$\boxed{0}$	0	1	7	$\cancel{0}$
5	16	$\boxed{0}$	3	4
$\cancel{0}$	4	3	$\boxed{0}$	1
4	$\boxed{0}$	$\cancel{0}$	2	$\cancel{0}$
2	2	7	9	$\boxed{0}$

No. of Assignment = Order of the matrix

∴ The optimum solution is

$A \rightarrow A'$, $B \rightarrow C'$, $C \rightarrow D'$, $D \rightarrow B'$, $E \rightarrow E'$

Assignment Value $= 10 + 6 + 4 + 9 + 10$

$=$ Rs. 39

Unbalanced Assignment Problem

1.

	A	B	C
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

Solution:

No. of rows $\neq$ No. of columns.

It is an unbalanced Assignment problem.

So, convert this into balanced, add a dummy column with all entries are zero.

$$\begin{matrix} 9 & 26 & 15 & 0 \\ 13 & 27 & 6 & 0 \\ 35 & 20 & 15 & 0 \\ 18 & 30 & 20 & 0 \end{matrix}$$

Reduced Row matrix:

$$\begin{matrix} 9 & 26 & 15 & 0 \\ 13 & 27 & 6 & 0 \\ 35 & 20 & 15 & 0 \\ 18 & 30 & 20 & 0 \end{matrix}$$

Reduced column matrix

$$\begin{matrix} \boxed{0} & 6 & 9 & \cancel{0} \\ 4 & 7 & \boxed{0} & \cancel{0} \\ 26 & \boxed{0} & 9 & \cancel{0} \\ 9 & 10 & 14 & \boxed{0} \end{matrix}$$

No. of Assignment = Order of the matrix

$\therefore$ The optimum solution is

$I \to A$, $II \to C$, $III \to B$.

Assignment value $= 9 + 6 + 20 = $ Rs. 35.

2.

	A	B	C	D
I	18	24	28	32
II	8	13	17	19
III	10	15	19	22

Solution:

No. of rows $\neq$ No. of Columns.

It is an unbalanced Assignment Problem.

So, convert this in to balanced, add a dummy row with all entries are zero.

$$\begin{matrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{matrix}$$

Reduced Row matrix:

$$\begin{matrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 4 & 12 \\ 0 & 0 & 0 & 0 \end{matrix}$$

Reduced column matrix

$$\begin{matrix} \boxed{0} & 6 & 10 & 14 \\ \cancel{0} & 5 & 9 & 11 \\ \cancel{0} & 5 & 4 & 12 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{matrix}$$

Iteration : 1

$$\begin{matrix} \boxed{0} & 2 & 6 & 10 \\ \cancel{0} & 1 & 5 & 7 \\ \cancel{0} & 1 & \boxed{0} & 8 \\ 4 & \boxed{0} & \cancel{0} & \cancel{0} \end{matrix}$$

Iteration : 2

$$\begin{matrix} \boxed{0} & 1 & 5 & 9 \\ \cancel{0} & \boxed{0} & 4 & 6 \\ 1 & 1 & \boxed{0} & 8 \\ 5 & \cancel{0} & \cancel{0} & \boxed{0} \end{matrix}$$

No. of Assignment = Order of the matrix.

∴ The optimum Solution is

$I \rightarrow A, \; II \rightarrow B, \; III \rightarrow C.$

Assignment value = $18 + 13 + 19$ = Rs. 50

Maximization Assignment Problem:

3.

	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

Solve the following assignment problem to find the maximum total expected sale.

Solution:

This is the maximization problem.

Convert this into minimization, subtract all the cost values from the maximum cost value 42

0	7	14	21
12	17	22	27
12	17	22	27
18	22	26	30

Reduced Row matrix

0	7	14	21
0	5	10	15
0	5	10	15
0	4	8	12

Reduced column matrix

$\boxed{0}$	3	6	9	✓
$\cancel{0}$	1	2	3	✓
$\cancel{0}$	1	2	3	✓
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	
✓				

Iteration: 1

$\boxed{0}$	2	5	8	✓
$\cancel{0}$	$\boxed{0}$	1	2	✓
$\cancel{0}$	$\cancel{0}$	1	2	✓
1	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	

Iteration : 2

$$\begin{matrix} \boxed{0} & 2 & 4 & 7 \\ \cancel{0} & \boxed{0} & \cancel{0} & 1 \\ \cancel{0} & \cancel{0} & \boxed{0} & 1 \\ 2 & 1 & \cancel{0} & \boxed{0} \end{matrix}$$

No of Assignment = Order of the matrix.

∴ The optimum solution is.

A → I , B → II , C → III , D → IV .

Assignment value = 42 + 25 + 20 + 12 = Rs. 99.

2.

	A	B	C	D
1	3	6	2	6
2	7	1	4	4
3	3	8	5	8
4	6	4	3	7
5	5	2	4	3
6	5	7	6	4

Solve the problem to maximize the total profit.

Solution :

This is maximization problem.

So, Convert this into minimization, subtract all the cost values from the maximum cost value 8 .

$$\begin{matrix} 5 & 2 & 6 & 2 \\ 1 & 7 & 4 & 4 \\ 5 & 0 & 3 & 0 \\ 2 & 4 & 5 & 1 \\ 3 & 6 & 4 & 5 \\ 3 & 1 & 2 & 4 \end{matrix}$$

No. of rows ≠ No. of columns.

This is unbalanced Assignment problem.

So, convert this in to balanced, add dummy columns with all the entries are zero.

$$\begin{array}{cccccc}
5 & 2 & 6 & 2 & 0 & 0 \\
1 & 7 & 4 & 4 & 0 & 0 \\
5 & 0 & 3 & 0 & 0 & 0 \\
2 & 4 & 5 & 1 & 0 & 0 \\
3 & 6 & 4 & 5 & 0 & 0 \\
3 & 1 & 2 & 4 & 0 & 0
\end{array}$$

<u>Reduced Row matrix</u>

$$\begin{array}{cccccc}
5 & 2 & 6 & 2 & 0 & 0 \\
1 & 7 & 4 & 4 & 0 & 0 \\
5 & 0 & 3 & 0 & 0 & 0 \\
2 & 4 & 5 & 1 & 0 & 0 \\
3 & 6 & 4 & 5 & 0 & 0 \\
3 & 1 & 2 & 4 & 0 & 0
\end{array}$$

<u>Reduced column matrix</u>

$$\begin{array}{cccccc}
4 & 2 & 4 & 2 & \boxed{0} & \cancel{0} \\
0 & 7 & 2 & 4 & \cancel{0} & \cancel{0} \\
4 & \boxed{0} & 1 & \cancel{0} & \cancel{0} & \cancel{0} \\
1 & 4 & 3 & 1 & \cancel{0} & \boxed{0} \\
2 & 6 & 2 & 5 & 0 & \cancel{0} \\
2 & 1 & \boxed{0} & 4 & \cancel{0} & 0
\end{array}$$

<u>Iteration: 1</u>

$$\begin{array}{cccccc}
3 & 1 & 3 & 1 & \boxed{0} & \cancel{0} \\
\boxed{0} & 7 & 2 & 4 & 1 & 1 \\
4 & \boxed{0} & 1 & \cancel{0} & 1 & 1 \\
\cancel{0} & 3 & 2 & \boxed{0} & \cancel{0} & \cancel{0} \\
1 & 5 & 1 & 4 & \cancel{0} & \boxed{0} \\
2 & 1 & \boxed{0} & 4 & 1 & 1
\end{array}$$

No. of Assignment = Order of the matrix

The optimum solution is

$2 \to A$, $3 \to B$, $4 \to D$

$6 \to C$.

Assignment value =

$7 + 8 + 7 + 6$

= Rs. 28

Home work:

1.

	A	B	C	D	E
I	32	38	40	28	40
II	40	24	28	21	36
III	41	27	33	30	37
IV	22	38	41	36	39
V	29	33	40	35	39

Find the maximum profit - Ans: 191

2.

	A	B	C	D	E
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

Find the maximum profit. Ans: 376

3.

Machines	Job I	II	III	IV	V	VI
1	9	22	58	11	19	27
2	43	78	72	50	63	48
3	41	28	91	37	45	33
4	74	42	27	49	39	32
5	36	11	57	22	25	18
6	13	56	53	31	17	28

Ans: 333

How should the jobs be assigned to the machines so as to maximize the over all return?

4.

	A	B	C	D
I	15	13	14	17
II	11	12	15	13
III	18	12	10	11
IV	15	17	14	16

Find the optimum solution.

Ans: 49

Transportation Problem

1. **Definition:**

The transportation problem is one of the subclasses of LPP's in which the objective is to transport various quantities of a single homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum

2. Mathematical formulation of the Transportation Problem:

A transportation problem can be ~~stored~~ stated mathematically as a linear programming problem as below:

$$\text{Minimize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}.$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2 \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2 \ldots n.$$

$$x_{ij} \geq 0, \quad \text{for all } \cancel{i \text{ and } j}. \ i \text{ and } j.$$

3. Existence of feasible **solution**:

A necessary and sufficient condition for the existence of a feasible solution to a transportation problem is that,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

4. Existence of optimum solution:

The feasible region of the problem is closed, bounded, and non-empty then there exists an optimum solution

5. Basic feasible solution :

The number of basic variables of the general transportation problem at any stage of feasible solution must be $m+n-1$

Non - Degenerate Basic feasible solution :

A feasible solution involving exactly $m+n-1$ positive variables is known as non-degenerate basic feasible solution.

Degenerate Basic feasible solution :

A feasible solution does not involving exac $m+n-1$ positive variables is known as degenerate basic feasible solution.

6. Balanced Transportation Problem :

~~Wheah~~ When the total demand is equal to total supply, the Transportation problem is said to be balanced

Unbalanced Transportation problem :

When the total demand is not equal to the total supply, the T.P is said to be unbalanced.

7. Occupied cells :

The allocated cells in the transportation table will be called occupied cells.

Non - Occupied cells :

The empty cells in the transportation table will be called non-occupied cells.

8. Transportation Table:

Destination

Origin	1	2	...	n	Supply
1	x_{11} c_{11}	x_{12} c_{12}	....	x_{1n} c_{1n}	
2	x_{21} c_{21}	x_{22} c_{22}	...	x_{2n} c_{2n}	
:	....	...		....	
m	x_{m1} c_{m1}	x_{m2} c_{m2}	...	x_{mn} c_{mn}	
Demand					

9. Loop:

In a transportation table, an ordered set of four or more cells is said to form a loop, if

(i) any two adjacent cells in the ordered set lie either in the same row or in the same column

(ii) any three (or) more adjacent cells in the ordered set donot lie in the same row or the same column.

The first cell of the set is considered to follow the last in the set

ie). Each cell must appear only once in the ordered set.

Note:

(i) Every loop has an even number of cells.

(ii) closed loops may or maynot be square in shape

Finding an Initial Basic feasible solution:

There are several methods available to obtain an initial basic feasible solution.

(i) North west corner method.

(ii) Least - cost method / Matrix Minima method.

iii) Vogel's Approximation method / VAM. / Penalty method

North West corner method:

It is a simple and efficient method to obtain an initial basic feasible solution various steps of the methods are

Step : 1

Select the north west (upper left hand) corner cell of the transportation table and allocation as much as possible so that either the capacity of the first row is exhausted or the destination requirement of the first column is satisfied. (ie). $x_{11} = \min(a_1, b_1)$

Step : 2

If $b_1 > a_1$, we move down vertically to the second row and make the second allocation of magnitude

$x_{21} = \min(a_2, b_1 - x_{11})$ in the cell $(2,1)$,

If $b_1 < a_1$, we move right horizontally to the second column and make the second allocation of magnitude

$x_{12} = \min(a_1 - x_{11}, b_2)$ in the cell $(1,2)$

If $b_1 = a_1$, there is a tie for the second column one can make the second allocation of magnitude

$x_{12} = \min(a_1 - a_1, b_1) = 0$ in the cell $(1,2)$

(or) $x_{21} = \min(a_2, b_1 - b_1) = 0$ in the cell $(2,1)$

Step : 3

Repeat steps 1 and 2 moving down towards the lower / right corner of the transportation table until all the Rim requirements are satisfied.

Least Cost Method (or) Matrix Minima Method

This method takes into account the minimum unit cost and can be summarized as follows.

Step : 1

Determine the smallest cost matrix of the transportation table. Let it be C_{ij} Allocate $x_{ij} = \min(a_i, b_j)$ in the cell (i, j)

Step : 2

If $x_{ij} = a_i$ cross off the i^{th} row of the transportation table and decrease b_j by a_i, Go to step 3

If $x_{ij} = b_j$ cross off the j^{th} column of the transportation table and decrease a_i by b_j, Go to step 3.

If $x_{ij} = a_i = b_j$ cross off either the i^{th} row or j^{th} column but not both.

Step : 3

Repeat the steps 1 & 2 for the resulting reduced transportation table until all the rim requirements are satisfied whenever the minimum cost is not unique, make an arbitrary choice among the minima

Vogel's Approximation Method [Penalty Method]

The vogel's Approximation method takes into account not only the least cost C_{ij} but also the costs that just exceed C_{ij}. The steps of the method are given below.

Step:1

For each row of the transportation table identify the smallest and the next to smallest cost. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly, Compute the differences for each column.

Step:2

Identify the row or column with the largest differences among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference Correspond to i^{th} row and let c_{ij} be the smallest cost in the i^{th} row. Allocate the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the $(i,j)^{th}$ cell and cross off either the i^{th} row or the j^{th} column in the usual manner.

Step:3

Recompute the column and row differences for the reduced transportation table and go to step 2, Repeat the procedure until all the rim requirements are satisfied.

MODI Method (Modified Distribution Method)

Transportation Algorithm

Various steps involved in solving any transportation problem may be summarized in the following iterative procedure.

Step : 1

Find the initial basic feasible solution by using any of the three methods like

(i) North west Corner
(ii) Least cost
(iii) Vogel's Approximation Method

Step : 2

check the number of occupied cells, If there are less than $m+n-1$, there exists degeneracy and we introduce a very small +ve assignment of $\epsilon\,(\simeq 0)$ in suitable independent positions, so that the number of occupied cells is exactly equal to $m+n-1$.

Step : 3

For each occupied cell in the current solution solve the system of equations $U_i + V_j = C_{ij}$ starting initially with some $U_i = 0$ (or) $V_j = 0$ and entering successively the values of U_i and V_j in the transportation table margins

Step : 4

Compute the net evaluations $Z_{ij} - C_{ij} = U_i + V_j - C_{ij}$ for all unoccupied basic cells and enter them in the lower left corners of the corresponding cells.

Step : 5

Examine the signs of each $Z_{ij} - C_{ij}$, If all $Z_{ij} - C_{ij} \leq 0$, then the current basic feasible solution is an optimum one. If atleast one $Z_{ij} - C_{ij} > 0$, select the unoccupied cell, having the largest ~~+ve~~ positive net evaluation to enter the basis.

Step: 6

Let the unoccupied cell (r, s) enter the basis. Allocate an unknown quantity, say θ, to the (r, s). Identify a loop that starts and ends at the cell (r, s) and connects some of the basic cells. Add and subtract interchangeably, θ to and from the transition cells of the loop in such a way that the rim requirement remain satisfied.

Step: 7

Assign a maximum value to θ in such a way that the value of one basic variable becomes zero and the other basic variables remain non-negative. The basic cell whose allocation has been reduced to zero leaves the basis.

Step: 8

Return to step 3 and repeat the process until an optimum basic feasible solution has been obtained.

Balanced Transportation Problem:

A necessary and sufficient condition for the existence of feasible solution to the general transportation problem is that the total demand must equal the total supply.

ie) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Unbalanced Transportation Problem:

Sometime there may be more demand than the supply and vice versa in which case the problem is said to be unbalanced $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$.

Alternative optimal Solution:

If an occupied cell in an optimal solution has opportunity cost of zero

i.e. $\Delta_{ij} = Z_{ij} - C_{ij} = 0$,

then an alternative optimal solution can be formed with another set of allocations without increasing the total transportation cost.

Maximization Transportation Problem:

In general, transportation method is used for Minimization problems. However it may also be used to solve problems in which objective to maximize when we consider the unit profit P_{ij} instead of unit cost C_{ij} associated with each route (i, j)

The solution procedure for solving such problems is summarized below

Step: 1

Convert the given problem into that of minimization by replacing each element of the transportation table by the difference from the maximum element of the table

Step: 2

Find an initial feasible solution using any of the three methods.

Step: 3

Use MODI method, for finding an optimum solution.

How to solve a unbalanced ~~Assignment~~ Transportation problem!

Given demand not equal to supply, this implies that the problem is unbalanced. Inorder to modify an unbalanced problem into a balanced problem, Consider a dummy source (or) destination which will provide the surplus supply (or) demand respectively. The costs of transporting an unit item form dummy sources are taken to be zero. Similarly, the costs of transporting an unit to dummy destinations are also taken to be zero

North-West Corner Rule:

Obtain the Initial Basic Feasible Solution to the following transportation problem:

1.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
	200	225	275	250	950

Solution:

$$\sum a_i = \sum b_j$$

It is a balanced Transportation Problem.

$\boxed{200}$ 11	13	17	14	50 ~~250~~
16	18	14	10	300
21	24	13	10	400
200	225	275	250	

$\boxed{200}$ 11	$\boxed{50}$ 13	17	14	~~50~~
16	18	14	10	300
21	24	13	10	400
	225 175	275	250	

$\boxed{175}$ 18	14	10	125 ~~300~~
24	13	10	400
175	275	250	

125 14	16	~~125~~
13	10	400
~~275~~ 150	250	

150 13	250 10	250 ~~400~~
~~150~~	~~250~~	

200 11	50 13	17	14
16	175 18	125 14	10
21	24	150 13	250 10

No. of allocations $= m+n-1$

$6 = 3+4-1$

$6 = 6$

$\therefore$ Non-Degenerate Basic feasible Solutions are

$x_{11} = 200$, $x_{12} = 50$, $x_{22} = 175$, $x_{23} = 125$.

$x_{33} = 150$, $x_{34} = 250$.

Transportation cost $= \sum C_{ij} x_{ij}$

$= 11 \times 200 + 13 \times 50 + 18 \times 175 + 14 \times 125$

$+ 13 \times 150 + 10 \times 250$

$=$ Rs. $12200/-$

2.

	D_1	D_2	D_3	D_4	Supply
S_1	3 3	2 7	6	4	5 2
S_2	2	1 4	1 3	2	2 1
S_3	4	3	1 8	2 5	3 2
Demand	~~3~~	3	~~2~~	2	10

Scanned by TapScanner

Soln :

$\sum a_i = \sum b_j$.

It is a balanced Transportation problem.

No. of allocations = $m+n-1$

$6 = 6$

∴ Non-Degenerate Basic feasible solutions are.

$x_{11} = 3$, $x_{12} = 2$, $x_{22} = 1$, $x_{23} = 1$, $x_{33} = 1$, $x_{34} = 2$.

Transportation cost $= \sum C_{ij} x_{ij}$

$= 3\times3 + 2\times7 + 1\times4 + 1\times3 + 1\times8 + 2\times5$

$=$ RS. 48/-

Least - Cost Method (Matrix - Minima Method).

1.

	D	E	F	G	Supply
A	$\boxed{200}$ 11	$\boxed{50}$ 13	17	14	250 50
B	16	$\boxed{175}$ 18	$\boxed{125}$ 14	10	300 175
C	21	24	$\boxed{150}$ 13	$\boxed{250}$ 10	~~400~~ 150
Demand.	200	225	275	250.	950

Solution.

$\sum a_i = \sum b_j$.

It is a balanced Transportation Problem.

No. of allocations = $m+n-1$.

$6 = 6$

∴ Non-Degenerate Basic feasible solutions are.

$x_{11} = 200$, $x_{12} = 50$, $x_{22} = 175$, $x_{23} = 125$, $x_{33} = 150$,

$x_{34} = 250$

Transportation cost $= \sum C_{ij} x_{ij}$

$= 11\times200 + 13\times50 + 18\times175 + 14\times12$

$+ 13\times150 + 10\times250$.

$=$ RS. 12200.

Scanned by TapScanner

2.

	D_1	D_2	D_3	D_4	
S_1	$\boxed{1}$ 3	7	$\boxed{2}$ 6	$\boxed{2}$ 4	~~5~~ ~~4~~
S_2	$\boxed{2}$ 2	4	3	2	~~2~~
S_3	4	$\boxed{3}$ 3	8	5	~~3~~
	~~3~~	~~3~~	~~2~~	~~2~~	

Solution:

$$\sum a_i = \sum b_j .$$

No. of allocations $= m+n-1$

$5 \neq 6$.

$\therefore$ Degenerate Basic feasible solutions are.

$x_{11} = 1$, $x_{13} = 2$, $x_{14} = 2$, $x_{21} = 2$, $x_{32} = 3$.

Transportation Cost $= \sum C_{ij} x_{ij}$

$= 1 \times 3 + 2 \times 6 + 2 \times 4 + 2 \times 2 + 3 \times 3$

$=$ Rs. 36.

Scanned by TapScanner

Find the IBFS for the following Transportation Problem by using (i) North west corner method (ii) Least cost method. (iii) Vogel's Approximation Method

1.

	A	B	C	Available
I	50	30	220	1
II	90	45	170	3
III	250	200	50	4
Supply	4	2	2	

Ans: a)

2.

	A	B	C	Available
I	6	8	4	14
II	4	9	8	12
III	1	2	6	5
Supply	6	10	15	

3.

	X	Y	Z	Available
A	8	7	3	60
B	3	8	9	70
C	11	3	5	80
Supply	50	80	80	

4.

7	3	4	2
2	1	3	3
3	4	6	5
4	1	5	

Vogel's Approximation Method :

1.

	D	E	F	G	Supply	Row Penalty I	II	III	IV
A	$\boxed{200}$ 11	$\boxed{50}$ 13	17	14	250 ~~50~~	(2)	(1)	–	–
B	16	$\boxed{175}$ 18	14	$\boxed{125}$ 10	300 125	(4)	(4)	(4)	(4)
C	21	24	$\boxed{275}$ 13	$\boxed{125}$ 10	400 275	(3)	(3)	(3)	(3)
Demand	200	225 175	275	250 125	950				

Column Penalty	D	E	F	G
I	(5)	(5)	(1)	(0)
II	–	(5)	(1)	(0)
III	–	(6)	(1)	(0)
IV	–	–	(1)	(0)

Solution :

$$\sum a_i = \sum b_j .$$

It is a balanced Transportation Problem.

No. of allocations = $m+n-1$

$6 = 6$.

∴ Non-Degenerate Basic feasible solutions are.

$x_{11} = 200$, $x_{12} = 50$, $x_{22} = 175$, $x_{24} = 125$, $x_{33} = 275$

$x_{34} = 125$

Transportation Cost = $\sum c_{ij} x_{ij}$.

$= 11 \times 200 + 13 \times 50 + 18 \times 175 + 10 \times 125$

$+ 13 \times 275 + 10 \times 125$

$=$ Rs. 12,075

2.

					Row Penalty I	II	III
$\boxed{3}$ 3	7	6	$\boxed{2}$ 4	~~5~~ 2	(1)	(1)	(1)
2	4	$\boxed{2}$ 3	2	~~2~~	(0)	–	(–)
4	$\boxed{3}$ 3	8	5	~~3~~	(1)	(1)	(–)
~~3~~	~~3~~	~~2~~	~~2~~	10			
I. (1)	(1)	(3)	(2)				
II (1)	(4)	–	(1)				

Column Penalty

Solution:

$$\sum a_i = \sum b_j .$$

It is a balanced Transportation Problem.

No. of allocations $= m+n-1$

$$4 \neq 6 .$$

∴ Degenerate Basic feasible Solutions are.

$x_{11} = 3$, $x_{14} = 2$, $x_{23} = 2$, $x_{32} = 3$.

Transportation Cost $= \sum C_{ij} x_{ij}$

$= 3 \times 3 + 2 \times 4 + 2 \times 3 + 3 \times 3 .$

∴ $=$ Rs. 32.

Scanned by TapScanner

MODI Method:

Solve the following Transportation Problem:

1.

				Supply	Row Penalty I	II	III	IV
21	16	25	[11] 13	~~11~~	(3)	–	–	–
[6] 17	[3] 18	14	[4] 23	~~13~~ ~~9~~ 3	(3)	(3)	(3)	(4)
32	[7] 27	[12] 18	41	~~19~~ ~~7~~	(9)	(9)	(9)	(9) ✓
~~6~~	10	~~12~~	~~4~~ ~~15~~	43				

Column Penalty

I	(4)	(2)	(4)	(10) ✓
II	(15)	(9)	(4)	(18) ✓
III	(15) ✓	(9)	(4)	–
IV	–	(9)	(4)	–

<u>Solution</u>: $\sum a_i = \sum b_j$.

It is a balanced Transportation Problem

No. of allocations = $m+n-1$

$6 = 6$.

This is Non-Degenerate Basic feasible solution.

-14 21	-8 16	-26 25	[11] 13	$u_1 = -10$
[6] 17	[3] 18	-5 14	[4] 23	$u_2 = 0$
-6 32	[7] 27	[12] 18	-9 41	$u_3 = 9$
$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$	

<u>For occupied cell</u>

$u_i + v_j = c_{ij}$

<u>For Unoccupied cell</u>

$\Delta_{ij} = (u_i + v_j) - c_{ij}$

All $\Delta_{ij} \leq 0$

∴ The optimum soln is

$x_{14} = 11,\ x_{21} = 6,\ x_{22} = 3,\ x_{24} = 4,\ x_{32} = 7,\ x_{33} = 12$

Transportation cost $= \sum c_{ij} x_{ij}$

$= 11 \times 13 + 6 \times 17 + 3 \times 18 + 4 \times 23 + 7 \times 27 + 12 \times 18$

$=$ Rs 796

2.

	D_1	D_2	D_3	D_4	Supply	Row Penalty I	II	III	IV
O_1	5	[18] 3	[1] 6	2	~~19~~ ✗	(1)	(2)	(2)✓	(1)
O_2	[12] 4	7	9	[25] 1	~~37~~ 12	(3)✓	(3)✓	–	–
O_3	[4] 3	4	[30] 7	5	~~34~~ ~~30~~	(1)	(1)	(1)	(4)✓
Demand	~~4~~ ~~16~~	~~18~~	~~31~~ ~~30~~	~~25~~	90				

Column Penalty

I	(1)	(1)	(1)	(1)
II	(1)	(1)	(1)	–
III	(2)	(1)	(1)	–
IV	(2)	–	(1)	

Solution:

$$\sum a_i = \sum b_j$$

It is a balanced T.P

No. of Allocations $= m+n-1$

$6 = 6$

This is Non_Degenerate Basic feasible solution.

-3 5	[18] 3	[1] 6	-3 2	$u_1 = 0$
[12] 4	-2 7	-1 9	[25] 1	$u_2 = 2$
[4] 3	0 4	[30] 7	-5 5	$u_3 = 1$
$v_1 = 2$	$v_2 = 3$	$v_3 = 6$	$v_4 = -1$	

All $\Delta_{ij} \leq 0$

$\therefore$ The optimum solution is

$x_{12} = 18$, $x_{13} = 1$, $x_{21} = 12$, $x_{24} = 25$, $x_{31} = 4$, $x_{33} = 30$.

Transportation cost $= \sum C_{ij} x_{ij}$

$= 18 \times 3 + 1 \times 6 + 12 \times 4 + 25 \times 1 + 4 \times 3 + 30 \times 7$

$=$ Rs. 355.

3.

				Supply
$\boxed{20}$ 1	2	$\boxed{10}$ 1	4	~~30~~ 10
8	$\boxed{20}$ 3	$\boxed{20}$ 2	$\boxed{10}$ 1	~~50~~ 40
4	$\boxed{20}$ 2	5	9	~~20~~
~~20~~	~~10~~ 40	~~30~~ 20	~~10~~	100

Row Penalty

I	II	III	IV
(0)	(0)	(0)	(1)✓
(1)	(1)	(1)	(1)
(2)	(2)✓	–	–

Column Penalty

I	(2)	(0)	(1)	(3)✓
II	(2)	(0)	(1)	–
III	(2)✓	(1)	(1)	–
IV	–	(1)	(1)	–

Solution

$$\Sigma a_i = \Sigma b_j$$

It is a balanced T.P.

No. of allocations $= m+n-1$

$6 = 6$

$\therefore$ This is a Non-Degenerate Basic feasible solution.

$\boxed{20}$ 1	0 / 2	$\boxed{10}$ 1	−4 / 4	$u_1 = -1$
−1 / 3	$\boxed{20}$ 3	$\boxed{20}$ 2	$\boxed{10}$ 1	$u_2 = 0$
−3 / 4	$\boxed{20}$ 2	−4 / 5	−9 / 9	$u_3 = -\phi$
$v_1 = 2$	$v_2 = 3$	$v_3 = 2$	$v_4 = 1$	

All $\Delta_{ij} \le 0$

$\therefore$ The optimum solution is

$x_{11} = 20,\ x_{13} = 10,\ x_{22} = 20,\ x_{23} = 20,\ x_{24} = 10,\ x_{32} = 20$

Transportation Cost $= \Sigma C_{ij} x_{ij}$

$= 1\times 20 + 1\times 10 + 3\times 20 + 2\times 20 + 1\times 10 + 2\times 20$

$=$ Rs. 180

4.

					Supply
	[5] 19	30	50	[2] 10	~~7~~ ~~2~~
	70	30	[7] 40	[2] 60	~~9~~
	40	[8] 8	70	[10] 20	~~18~~ ~~10~~
Demand	~~5~~	~~8~~	~~7~~	~~14~~ ~~4~~ ~~2~~	34

Row Penalty

I	II	III	IV
(9)	(9)	(40)	(40)
(10)	(20)	(20)	(20)
(12)	(20)	(50) ✓	–

Column Penalty

I	(21)	(22) ✓	(10)	(10)
II	(21) ✓	–	(10)	(10)
III	–	–	(10)	(10)
IV	–	–	(10)	(50) ✓

<u>Solution:</u>

$$\Sigma a_i = \Sigma b_j .$$

It is a balanced Transportation Problem.

No. of allocations = $m+n-1$

$6 = 6$

$\therefore$ This is a Non-Degenerate Basic feasible soln.

[5] 19	-32 30	-66 50	[2] 10	$U_1 = 10$
-1 70	18 30 +	[7] 40	[2] 60 −	$U_2 = 60$
-11 40	[8] 8 −	-10 70	[10] 20 +	$U_3 = 20$
$V_1 = 9$	$V_2 = -12$	$V_3 = -20$	$V_4 = 0$	

Iteration : 1.

[5] 19	-32 30	-42 50	[2] 10	$U_1 = 0$
-19 70	[2] 30	[7] 40	-18 60	$U_2 = 32$
-11 40	[6] 8	-52 70	[12] 20	$U_3 = 10$
$V_1 = 19$	$V_2 = -2$	$V_3 = 8$	$V_4 = 10$	

All $\Delta_{ij} \leq 0$.

$\therefore$ The optimum solution is.

$x_{11} = 5$, $x_{14} = 2$, $x_{22} = 2$, $x_{23} = 7$, $x_{32} = 6$, $x_{34} = 12$

Transportation cost $= \sum C_{ij} x_{ij}$

$= 19 \times 5 + 10 \times 2 + 30 \times 2 + 40 \times 7$
$+ 8 \times 6 + 20 \times 12$

$=$ Rs. 743.

5. Consider a transportation problem with $m = 3$, $n = 4$ where,

$C_{11} = 2$, $C_{12} = 3$, $C_{13} = 11$, $C_{14} = 7$.

$C_{21} = 1$, $C_{22} = 0$, $C_{23} = 6$, $C_{24} = 1$

$C_{31} = 5$, $C_{32} = 8$, $C_{33} = 15$, $C_{34} = 9$

Suppose $S_1 = 6$, $S_2 = 1$, $S_3 = 10$ where as $D_1 = 7$, $D_2 = 5$, $D_3 = 3$, and $D_4 = 2$. Apply the transportation simplex method to find an optimal solution

					Supply	Row Penalty I	II	III
	$\boxed{1}$ 2	$\boxed{5}$ 3	11	7	~~1~~ ~~6~~	(1)	(1)	(5) ✓
	1	0	6	$\boxed{1}$ 1	~~1~~	(1)	–	–
	$\boxed{6}$ 5	8	$\boxed{3}$ 15	$\boxed{1}$ 9	~~4~~ ~~8~~ ~~10~~	(3)	(3)	(4).
Demand	~~6~~ ~~7~~	~~5~~	~~3~~	~~1~~ ~~2~~	17			
Column penalty I.	(1)	(3)	(5)	(6) ✓				
II	(3)	(5) ✓	(4)	(2).				
III.	(3)	–	(4)	(2).				

Solution:

$\sum a_i = \sum b_j$.

It is a balanced T.P

No. of allocations $= m + n - 1$

$6 = 6$.

Scanned by TapScanner

This is non degenerate basic feasible soln.

[1] 2	[5] 3	11	-1 7	$u_1 = -2$
-4 1	0	6	[1] 1	$u_2 = -8$
[6] 5	8	[3] 15	[1] 9	$u_3 = 0$
$v_1 = 5$	$v_2 = 6$	$v_3 = 15$	$v_4 = 9$	

-1 2	[5] 3	[1] 11	-2 7	$u_1 = -4$
-11 1	-1 0	4 6	[1] 1	$u_2 = -8$
[7] 5	-1 8	[2] 15	[1] 9	$u_3 = 0$
$v_1 = 5$	$v_2 = 7$	$v_3 = 15$	$v_4 = 9$	

-1 2	[5] 3	[1] 11	-2 7	$u_1 = -4$
-5 1	-2 0	[1] 6	-1 1	$u_2 = -9$
[7] 5	-1 8	[1] 15	[2] 9	$u_3 = 0$
$v_1 = 5$	$v_2 = 7$	$v_3 = 15$	$v_4 = 9$	

All $\Delta_{ij} \leq 0$.

$\therefore$ The optimum soln is

$x_{12} = 5, \quad x_{13} = 1, \quad x_{23} = 1, \quad x_{31} = 7, \quad x_{33} = 1, \quad x_{34} = 2$.

Transportation cost $= \sum C_{ij} x_{ij}$

$= 3\times5 + 11\times1 + 6\times1 + 5\times7 + 15\times1 + 9$

$=$ Rs. 100.

6

6	1	9	3	70
11	5	2	8	55
10	12	4	7	90
85	35	50	45	215

Given, $x_{13} = 50$, $x_{14} = 20$, $x_{21} = 55$

$x_{31} = 30$, $x_{32} = 35$, $x_{34} = 25$

Solve the T.P.

Soln:

0 6	7 1	[50] 9 −	[20] 3 +	$u_1 = -4$
[55] − 11	8 5	12 2 +	0 8	$u_2 = 1$
[30] + 10	[35] 12	9 4	[25] 7 −	$u_3 = 0$
$V_1 = 10$	$V_2 = 12$	$V_3 = 13$	$V_4 = 7$	

max (7, 8, 12, 9)
= 12

Iteration: 1

12 6	19 + 1	[25] 9 −	[45] 3	$u_1 = 7$
[30] − 11	8 5	[25] 2 +	−12 8	$u_2 = 0$
[55] + 10	[35] 12 −	−3 4	−12 7	$u_3 = -1$
$V_1 = 11$	$V_2 = 13$	$V_3 = 2$	$V_4 = -4$	

Iteration: 2

−7 6	[25] 1	−19 9	[45] 3	$u_1 = -12$
[5] − 11	8 + 5	[50] 2	7 8	$u_2 = 0$
[80] + 10	[10] 12 −	−3 4	7 7	$u_3 = -1$
$V_1 = 11$	$V_2 = 13$	$V_3 = 2$	$V_4 = 15$	

Iteration : 3

(-7) 6	[25] 1 +	(-11) 9	[45] 3 −	$u_1 = 1$
(-8) 11	[5] 5	[50] 2	(-1) 8	$u_2 = 5$
[85] 10	[5] 12 −	(5) 4	(7) 7 +	$u_3 = 12$.

$V_1 = -2$ $V_2 = 0$ $V_3 = -3$ $V_4 = 2$

Iteration : 4

(0) 6	[30] 1	(-11) 9	[40] 3	$u_1 = 0$
(-1) 11	[5] 5	[50] 2	(-1) 8	$u_2 = 4$
[85] 10	(-7) 12	(-2) 4	[5] 7	$u_3 = 4$

$V_1 = 6$ $V_2 = 1$ $V_3 = -2$ $V_4 = 3$

All $\Delta_{ij} \le 0$

The optimum soln is

$x_{12} = 30$, $x_{14} = 40$, $x_{22} = 5$, $x_{23} = 50$, $x_{31} = 85$, $x_{34} = 5$

Transportation Cost $= \sum c_{ij} x_{ij}$

$= 1 \times 30 + 3 \times 40 + 5 \times 5 + 2 \times 50 + 10 \times 85 + 7 \times 5$

$=$ Rs. 1160.

7.

				Row Penalty I	Row Penalty II
[1] 50	80	220	1	(20)	(20)
[3] 90	45	170	3	(45)	(45)
250	[2] 200	[2] 50	~~4~~ 2	(150) ✓	(50) ✓
4	~~2~~	~~2~~	8		
Column penalty I	(40)	(15)	(120)		
Column penalty II	(40)	(15)	–		

Solution:

$$\sum a_i = \sum b_j$$

It is a balanced T.P.

No of allocations = $m+n-1$

$4 \neq 5$

∴ This is degenerate basic feasible soln.

$\boxed{1}$ 50	-30 30	-370 220	$u_1 = -200$
$\boxed{3}$ 90	-5 45	-280 170	$u_2 = -160$
$\boxed{\epsilon}$ 250	$\boxed{2}$ 200	$\boxed{2}$ 50	$u_3 = 0$
$V_1 = 250$	$V_2 = 200$	$V_3 = 50$	

All $\Delta_{ij} \le 0$.

∴ The optimum soln is

$x_{11} = 1,\ x_{21} = 3,\ x_{32} = 2,\ x_{33} = 2$.

Transportation cost $= \sum C_{ij} x_{ij}$

$= 1 \times 50 + 3 \times 90 + 2 \times 200 + 2 \times 50$

$=$ Rs 820.

8

								Row Penalty I	II	III	IV	V	VI
	4	$\boxed{20}$ 5	7	7	5	$\boxed{40}$ 3	~~40~~ ~~60~~	(2)	(2)	(2)	(4) ✓	–	–
	$\boxed{10}$ 9	11	$\boxed{10}$ 6	11	–	5	10 ~~20~~	(1)	(1)	(1)	(1)	(3)	(3) ✓
	11	10	$\boxed{30}$ 6	$\boxed{20}$ 2	$\boxed{40}$ 2	8	30 70 ~~90~~	(0)	(0)	(4) ✓	(2)	(5) ✓	–
	$\boxed{50}$ 9	10	9	6	9	12	50	(3)	(3)	(0)	(0)	(0)	(0)
	60	~~20~~	~~40~~ 10	~~20~~	~~40~~	~~40~~	220						
Column penalty I	(2)	(5) ✓	(0)	(4)	(3)	(2)							
II	(2)	–	(0)	(4) ✓	(3)	(2)							
III	(2)	–	(0)	–	(3)	(2)							
IV	(2)	–	(0)	–	–	(2)							
V	(2)	–	(0)	–	–	–							
VI	(0)	–	(3)	–	–	–							

$\sum a_i = \sum b_j$

it is a balanced T.P.

No of allocation = m+n-1

= 4+6-1

= 9

This is degenerate basic feasible soln.

[illegible]	$\boxed{20}$ 3	7	7	5	$\boxed{40}$ 3	$u_1 = -2$
$\boxed{10}$ 2	[illegible]	$\boxed{10}$ 2	[illegible]	[illegible]	5	$u_2 = 0$
[illegible]	2	$\boxed{30}$ 6	$\boxed{20}$ 2	$\boxed{40}$ 2	8	$u_3 = 0$
$\boxed{50}$ 0	0	4	6	4	12	$u_4 = 0$

$v_1 = 0$ $v_2 = 7$ $v_3 = 6$ $v_4 = 2$ $v_5 = 2$ $v_6 = 5$

All $\Delta_{ij} \le 0$

The optimum solution is

$x_{12} = 20$, $x_{16} = 40$, $x_{21} = 0$, $x_{23} = 10$

$x_{33} = 30$, $x_{34} = 20$, $x_{35} = 40$, $x_{41} = 50$

Transportation cost $= \sum C_{ij} x_{ij}$

$= 5 \times 20 + 3 \times 40 + 4 \times 10 + 6 \times 10$

$+ 6 \times 30 + 2 \times 20 + 2 \times 40 + 5 \times 50$

= Rs. 1120.

Unbalanced Transportation Problem:

1.

5	1	7	10
6	4	6	80
3	2	5	5
75	20	50.	

Soln:

$\sum a_i \neq \sum b_j$. (ie) $95 \neq 145$

It is an unbalanced T.P.

So, we add a dummy row with all cost values are zero.

				Row Penalty - I	II	III
5	$\boxed{10}$ 1	7	~~10~~	(4)	(4)✓	–
$\boxed{70}$ 6	$\boxed{10}$ 4	6	~~80~~	(2)	(2)	(2)
$\boxed{5}$ 3	2	5	~~5~~	(1)	(1)	(1)
0	0	$\boxed{50}$ 0	~~50~~	(0)	~~(0)~~	–
70 ~~75~~	10 ~~20~~	~~50~~	145			

Column Penalty			
I.	(3)	(1)	(5)✓
II	(2)	(1)	–
III	(3)✓	(2)	–

No. of allocations = $m+n-1$

$5 = 4+3-1$

$5 \neq 6$.

Degenerate Basic feasible solution.

Scanned by TapScanner

−2 5	$\boxed{10}$ 1	−4 7	$u_1 = -3$
$\boxed{70}$ 6	$\boxed{10}$ 4	$\boxed{\epsilon}$ 6	$u_2 = 0$
$\boxed{5}$ 3	−1 2	−2 5	$u_3 = -3$
0 0	−2 0	$\boxed{50}$ 0	$u_4 = -6$

$v_1 = 6$ $v_2 = 4$ $v_3 = 6$

All $\Delta_{ij} \leq 0$.

The optimum soln is.

$x_{12} = 10$, $x_{21} = 70$, $x_{22} = 10$, $x_{31} = 5$.

Transportation Cost $= \sum c_{ij} x_{ij}$

$= 1 \times 10 + 6 \times 70 + 4 \times 40 + 3 \times 5$

$=$ Rs. 605

110
420
160
15
105

2)

7	10	14	8	30
7	12	12	6	40
5	8	15	9	50
20	20	25	30.	

Solution:

$\sum a_i \neq \sum b_j$.   (i). $120 \neq 95$

It is an unbalanced T.P.

So, Convert this into balanced, add dummy column with all cost values are zero.

					Supply	Row Penalty I	II	III	IV
7	10	[5] 14	8	[25] 0	~~30~~ 5	(7)	(1)	(2)	(4)
7	12	[10] 12	[30] 6	0	~~40~~ 10	(6)	(1)	(6)	(0)
[20] 5	[20] 8	[10] 15	9	0	~~50~~ ~~30~~ 10	(5)	(3)	(1)	(7)
~~20~~	~~20~~	~~25~~ ~~15~~ 10	~~30~~	~~25~~					

Column Penalty

I	(2)	(2)	(2)	(2)	(0)
II	(2)	(2)	(2)	(2)	–
III	–	(2)	(2)	(2)	–
IV	–	(2)	(2)	–	–

No. of allocations = $m+n-1$

$7 = 3+5-1$

$7 = 7$

Non-Degenerate Basic feasible soln

-3 7	-3 10	[5] 14 −	0 8	[25] 0 −	$u_1 = -1$
-5 7	-7 12	[10] 12	[30] 6	-2 0	$u_2 = -3$
[20] 5	[20] 8	[10] 15 −	0 9	1 0 +	$u_3 = 0$

$V_1 = 5$ $V_2 = 8$ $V_3 = 15$ $V_4 = 9$ $V_5 = 1$

-2 7	-2 10	[15] 14	0 8	[15] 0	$u_1 = 0$
-4 7	-6 12	[10] 12	[30] 6	-2 0	$u_2 = -2$
[20] 5	[20] 8	-1 15	-1 9	[10] 0	$u_3 = 0$

$V_1 = 5$ $V_2 = 8$ $V_3 = 14$ $V_4 = 8$ $V_5 = 0$

All $\Delta_{ij} \le 0$.

The optimum soln is

$x_{13} = 15, \quad x_{23} = 10, \quad x_{24} = 30.$

$x_{31} = 20, \quad x_{32} = 20.$

Transportation cost $= \sum c_{ij} x_{ij}$

$= 14 \times 15 + 12 \times 10 + 6 \times 30$

$+ 5 \times 20 + 8 \times 20$

$=$ Rs. 770

Maximization Problem:

1. A departmental store wishes to stock the following quantities of a proper popular product in three types of container:

Container type:	1	2	3
Quantity:	170	200	180

Tenders are submitted by four dealers who undertake to supply not more than the quantities shown below:

Dealer:	1	2	3	4
Quantity:	150	160	110	130

The store estimates that profit per unit will vary with dealer as shown below:

	Dealer			
Container type	1	2	3	4
1	8	9	6	3
2	6	11	5	10
3	3	8	7	9

Soln:

This is maximization Problem.

So, Convert this in to minimization, subtract all the cost values from the maximum cost value 11.

				Supply	Row penalty I	II	III	IV
[150] 3	[20] 2	5	8	~~170~~ ~~20~~	(1)	(3)	—	—
5	[140] 0	6	[60] 1	~~200~~ ~~60~~	(1)	(1)	(1)	(5)
8	3	[110] 4	[70] 2	~~180~~ ~~110~~	(1)	(1)	(1)	(2)
~~150~~	~~160~~ ~~140~~	110	~~130~~ 70	550				

Column Penalty

I	(2)	(2)	(1)	(1)
II	—	(2)	(1)	(1)
III	—	(3)	(2)	(1)
IV	—	—	(2)	(1)

$$\Sigma a_i = \Sigma b_j$$

It is a balanced T.P

No. of allocations $= m+n-1$

$6 = 3+4-1$

$6 = 6$.

Non-Degenerate Basic feasible solution

[150] 3	[20] 2	0 5	-5 8	$u_1 = 0$
-4 5	[140] 0	-3 6	[60] 1	$u_2 = -2$
-6 8	-2 3	[110] 4	[70] 2	$u_3 = -1$
$v_1 = 3$	$v_2 = 2$	$v_3 = 5$	$v_4 = 3$	

All $\delta_{ij} \leq 0$.

The optimum soln is

$x_{11} = 150$, $x_{12} = 20$, $x_{22} = 140$, $x_{24} = 60$.

$x_{33} = 110$, $x_{34} = 70$.

Transportation Cost $= \sum c_{ij} x_{ij}$

$= 8 \times 150 + 9 \times 20 + 11 \times 140$

$+ 10 \times 60 + 7 \times 110 + 9 \times 70$

$=$ Rs. 4920.

Home work :

Problem 4. Solve the following Transportation Problem :

1.

6	4	1	5	14
8	9	2	7	16
4	3	6	2.	5
6	10	15	4.	

2.

7	9	3	2	16
4	4	3	5	14
6	4	5	8	20.
11	9	22	8 .	

3 .

5	2	4	3	22
4	8	1	6	15
4	6	7	5	8 .
7	12	17	9.	

42	48	35	37	[illegible]
40	49	52	51	150
37	38	40	43	190
80	90	140	160	

5

8	6	5	150
6	6	6	150
10	8	4	150
8	6	4	150
200	200	200	

Find the maximum profit case.