Discrete Mathematics

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Unit - I

Statements and Notation :-

statements :-

All the declarative sentences to which it is possible to assign one and only one of the two possible truth values are called statements.

These statements which do not contain any of the connectives are called atomic (primary, primitive) statements.

Example of statements Truth value

1. Canada is a country → T
   ∴ This is a statement
2. Moscow is the capital of Spain → F
   ∴ This is a statment
3. This statement is false → not a statement.
   Because Truth value T → F
   F → T } → semantic paradox.
4. 1 + 101 = 110. → Statement depends the nature
   Binary → True
   Decimal → False
5. close the door → it is a command
6. Toronto is an old city
   → statement

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7. Man will reach Mars by 1980.
↳ Statement.

1.2 Connectives:-

1.2.1. Negation:-

The negation of a statement is generally formed by introducing the word "not" at a proper place in the statement or by prefixing the statement with the pharse "It is not the case that." If 'p' denotes a statement, then the negation of 'p' is written as "$\neg p$" and read as "not p".
If the truth value of "p' is T then the truth value of "$\neg p$" is F.
If the truth value of "p" is F then the truth value of "$\neg p$" is T.

Truth Table

P	$\neg P$
T	F
F	T

Ex:- Consider the statement,
P: London is a city

Then $\neg P$: It is not the case that, London is a city

(or) $\neg P$: London is not a city.

Remark :-

Alternate symbol used for "$\neg$" are "$\sim$ (or) a bar) (or) " NOT ".

ie, $\neg P$ (or) $\sim P$, $\bar{P}$ (or) NOT P.

1.2.2 Conjunction :-

The conjunction of two statements P and Q is the statement $P \wedge Q$ which is read as "P and Q". The statement $P \wedge Q$ has the truth value T whenever both P and Q have the truth value T. otherwise it has the truth value F.

Truth Table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

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Ex:- (1) P : It is raining today
Q : There are 20 tables in this room.

$P \wedge Q$: It is raining today and there are 20 tables in this room.

(2) Translate into symbolic form :-

Jack and Jill went up the hill.

let
P : Jack went up the hill
Q : Jill went up the hill.

Symbol form is $P \wedge Q$.

1.2.3 Disjunction

The disjunction of two statements P and Q is the statement $P \vee Q$ which is read as "P or Q". The statement $P \vee Q$ has the truth value F only both P and Q have the truth value F; otherwise it is true. This disjunction is defined by

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

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1.2.4. Statement Formulas and Truth Tables

* Statements which do not contain any connectives are called "atomic" or "primary" or "Simple" statements
Ex, P, and Q

* Statements which contain one or more primary statements and some connectives are called "molecular" ~~and~~ or "composite" or "compound" statements.
Ex; $\neg P$, $P \vee Q$, $P \wedge Q$, $(P \vee \neg Q) \wedge P$.

<u>Statement</u> Formula :-
The Compound statements are the statement formulas which are derived from the statement Variables.

Ex: 1 Construct Truth table for $P \vee \neg Q$.

Ans:

P	Q	$\neg Q$	$P \vee \neg Q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

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Ex: 2 Construct the Truth table for $P \wedge \neg P$

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

Ex 3: Construct the T.T. for $(P \vee Q) \vee \neg P$

P	Q	$P \vee Q$	$\neg P$	$(P \vee Q) \neg P$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

1.2.6 Conditional and BiConditional

Conditional:-

If P and Q are any two statements, then the statement $P \to Q$ which is read as "If P then Q" is called a Conditional statement. The statement $P \to Q$ has a truth value F when Q has the truth value F and P has the truth value T; otherwise it has the truth value T.

The Conditional is defied by

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

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* The statement P is called antecedent and Q is called Consequent in $P \to Q$.

Ex: 1. Express in English the statement $P \to Q$, where

P : The Sun is shining today

Q : $2+7>4$.

Soln:-

If the sun is shining today then $2+7>4$.

* $P \to Q$ also represent to any one of the following

1. Q is necessary for P
2. P is Sufficient for Q.
3. Q if P
4. P only if Q.
5. P implies Q.

Ex 2:- write the following statement in Symbolic form: If either Jerry takes calculus or Ken takes Sociology then Larry will take English

Soln:- Let J : Jerry takes calculus

K : Ken takes Sociology

L : Larry takes English

Ans : $(J \vee K) \to L$.

Ex3:- Write in symbolic form the statement
The crop will be destroyed if there is a flood.

Soln:-

Let C: The crop will be destroyed

F: There is a flood.

This is in the form of C if F

$\therefore$ $F \rightarrow C$

Ex4:- Construct the T.T. for $(P \rightarrow Q) \wedge (Q \rightarrow P)$

Soln:-

P	Q	$P \rightarrow Q$	Q	P	$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	T	F	F	F
F	T	T	F	F	T	T

Biconditional

If P and Q are any two statements then the statement $P \rightleftarrows Q$ which is read as "P if and only if Q" is called a Biconditional statement. The statement $P \rightleftarrows Q$ has the truth value T whenever both P and Q have identical truth values.

The Biconditional is defined by,

P	Q	$P \rightleftarrows Q$
T	T	T
T	F	F
F	T	F
F	F	T

* $P \rightleftarrows Q$ is also read as " P is necessary and sufficient for Q".

<u>Exam 5</u> :- Construct the T.T for the formula

$$\neg(P \wedge Q) \rightleftarrows (\neg P \vee \neg Q)$$

<u>Soln</u> :-

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$ ✓	$\neg P \vee \neg Q$ ✓	$\neg(P \wedge Q) \rightleftarrows (\neg P \vee \neg Q)$
T	~~F~~T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

1.2.7. Well-formed Formulas (WFF):-

A statement formula is an expression which is a string consisting of variables (capital letters with or without subscripts), parenthesis, and connective symbols.

A well-formed formula can be generated by the following rules:

1. A statement variable standing alone is a well-formed formula
2. If A is a well-formed formula then $\neg A$ is a well-formed formula.
3. If A and B are well-formed formulas then $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$ and $(A \rightleftarrows B)$ are well-formed formulas.
4. A string of symbols containing the statement variables, connectives and parentheses is a well-formed formula iff it can be obtained by finitely many applications of the rules 1, 2, and 3.

Examples:-

$\neg(P \wedge Q)$, $\neg(P \vee Q)$, $(P \rightarrow (P \vee Q))$

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1.2.8. Tautologies :-

Tautology :- A Statement formula which is true regrdless of the truth values of the Statements which replace the Variables in it is called a "Universally Valid formula" or "~~Taug~~ "Tautology" or ~~a~~ a "logical Truth".

Ex:-

P	¬P	P ∨ ¬P
T	F	T
F	F	T

The formula $P \vee \neg P$ is Tautology.

* **Contradiction** :-
A statement formula which is False regardless of the truth values of the Statements which replace the Variables in it is called a "Contradiction" or "Identically False".

* The negation of a contradiction is a Tautology.

Ex:-

P	¬P	P ∧ ¬P
T	F	F
F	T	F

The formula $P \wedge \neg P$ is a Contradiction.