CHAPTER VIII
Centre of Gravity

§ 1. Centre of like parallel froces:

Let P_1, P_2, P_3 be a set of like parallel forces acting at points A_1, A_2, A_3 of a rigid body. The two forces P_1, P_2 will have a resultant $(P_1 + P_2)$, parallel to P_1 or P_2 acting at the point C_1 on A_1A_2 such that

centre point.

$$\frac{A_1C_1}{C_1A_2} = \frac{P_2}{P_1}$$

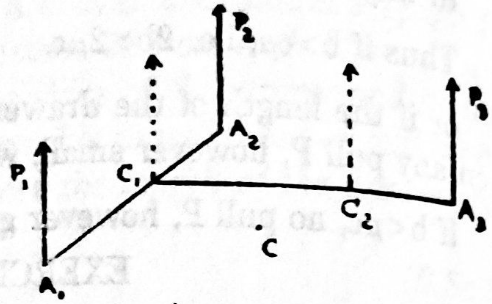


Fig. 1

Taking the forces $(P_1 + P_2)$ at C_1 and P_3 at A_3, their resultant is $(P_1 + P_2 + P_3)$, parallel to the parallel forces, acting at the point C_2 on C_1A_3 such that $\frac{C_1C_2}{C_2A_3} = \frac{P_3}{P_1+P_2}$

Proceeding in this manner till all the given forces are exhausted, we see that the resultant of all the parallel forces is a single force $(P_1 + P_2 + P_3 + ...)$ acting in the same direction as the component forces and passing through a point C.

It is clear that the positions of C_1, C_2, C_3etc. depend only on the magnitudes of $P_1, P_2, P_3, ...$ and on the positions of the points A_1, A_2, A_3 ... where they act and not at all with the common directions of the parallel forces. Hence the point C arrived at through which the final resultant of the parallel forces passes, is fixed, whatever be the common direction of the parallel forces, so long as their magnitudes and points of application remain unchanged. This fixed point C is defined as the *centre of the given system of like parallel forces.*

It should be noted that we will arrive at the same point C finally, in whatever order we proceed to combine the given parallel forces in succession.

Hence *the centre of parallel forces is a unique point.*

sm CT?

§ 2 Centre of Mass or Centre of Inertia:

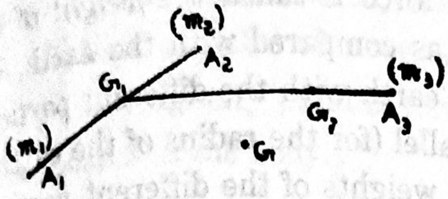


Fig. 2.

Let a system of rigidly connected particles of masses $m_1, m_2, m_3, \dots$ be placed at the points $A_1, A_2, A_3, \dots$ Suppose these particles are acted on by like parallel forces, the forceon each particle being proportional to its mass. The resultant of forces m_1, m_2 is a force $m_1 + m_2$ parallel to $m_1$orm_2 acting at the point G_1 on A_1A_2 such that

$$\frac{A_1G_1}{G_1A_2} = \frac{m_2}{m_1}$$

Taking the forces $(m_1 + m_2)$ at G_1 and m_3 at A_3, their resultant is $(m_1 + m_2 + m_3)$, parallel to m_1 acting at the point G_2 on G_1A_3 such that

$$\frac{G_1G_2}{G_2A_3} = \frac{m_3}{m_1 + m_2}$$

Proceeding thus, when all the particles have been exhausted, we arrive at a final point G.

This point is called *the centre of mass, or centre of inertia of the given system of particles.*

Clearly the point G is identical with the centre of a set of like parallel forces proporitonal to m_1, m_2, m_3 etc acting at A_1, A_2, A_3 etc. and hence as proved in § 2, it is a unique point.

Instead of a system of particles, we have a rigid body of any shape. In that case, the centre of mass is defined as follows: We can divide the body into a large number of indefinitely small elements. On these small elements, we can suppose a system of like parallel forces to act, the force acting on each element being proportional to its mass. The resultant of these parallel forces will pass through a definite point in the body, which is called its centre of mass.

§ 3. Centre of Gravity:

A rigid body may be considered as a collection of an inflinte number of particles rigidly connected with one another. On account of the attraction of the earth, every particle of the body is attracted towards the centre of the earth with a force proportional to the mass of particle. This force is called the *weight* of the particle. If the body is small as compared with the earth, the lines joining the centre of the earth with the different particles of the body will be almost parallel (for the radius of the earth is about 4000 miles). Hence the weights of the different particles of the body form a system of parallel forces. The resultant of these like parallel forces is called the *weight of the body* and is equal to the sum of the weights of the component particles. It will pass through a definite point of the body. This point is called the *Centre of Gravity or Centre of Mass* of the body.

Definition: *The centre of gravity of a body is that point through which the line of action of the weight of the body always passes in whichever position the body is held.* 2m

The centre of gravity of a body is a point fixed relative to it or rigidly connected with it. It may or may not lie in the body itself. Thus the centre of a gravity of a circular ring is its centre, which is not a point of the ring.

Centre of gravity is denoted by the letters C. G.

§ 4. Distinction between centre of gravity and centre of mass.

A body has a centre of gravity only (1) when it has weight i.e. it is acted upon by gravity and (2) when the weights of the particles of the body form a system of parallel forces, proportional to the masses of these particles. These two conditions must be satisfied, in order that a body may have a centre of gravity.

Suppose the body is imagined to be removed to an infinite distance in space, away from the earth, so that the earth does not have atraction on the body. Or, suppose the body to be taken to the centre of the earth, where also the attraction of the earth is zero. In these cases, the body will have no weight and there will be no centre of gravity. But the body will always have its mass which is independent of the earth's attraction on it and which is an inherent property of the material of the body. Hence it will always have a centre of mass.

Again, if the dimensions of the body are so large that the lines joining its various points to the centre of the earth cannot be supposed to be parallel, the body cannot have a centre of gravity. But it will have a centre of mass, for this point is the centre of a number of imaginary parallel forces supposed to be acting at different points of the body.

For a body of any shape, the centre of mass always concides with its centre of gravity, if the latter exists. Thus a body having its centre of gravity has its centre of mass, but one having its centre of mass may or may not have its centre of gravity.

To sum up,

(1) A rigid body, small in size compared with the earth and situated on or near its surface, has a centre of gravity.

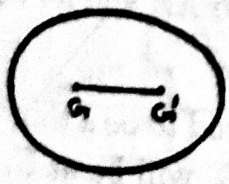
(2) A very large body may or may not have a centre of gravity but it has a centre of mass.

(3) A body outside the sphere of earth's attraction (it may be either at a very large distance from the earth or it may be at the centre of the earth) has no centre gravity but it has always a centre of mass.

(4) The centre of graity of a body, if it exists, always coincides with its centre of mass.

§ 5. The centre of gravity of a body is unique:

We can show that a body can have only one centre of gravity.



For, if possible, let it have two centres of gravity, G and G′. This means that, in all positions of the body, its weight acts through G as well as G^1. Hence the weight acts along GG′.

Fig. 3.

Now, hold the body such that GG' is horizontal. In this position, the weight which is a vertically downward force acts along a horizontal line GG', which is absurd. Hence the body cannot have two centres of gravity. i.e. the C.G. of the body is unique.

§ 6. Determination of the centre of gravity in simple cases:

1. C. G. of a thin uniform rod:

Let AB be a thin uniform rod of any material and G its midpoint. Consider two small equal lengths PP' and QQ' of the rod such that $GP = GQ$.

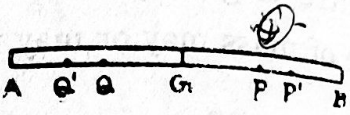


Fig. 4

C.G

The weights of these elements are equal and can be considered as two like parallel forces acting at P and Q. The resultant of these two equal like parallel forces acts at the middle point of PQ i.e, at G.

The whole rod can be divided into pairs of such equal elements equidistant from G. For each pair, the resultant of the weights acts at G. Hence the weight of the whole rod acts at G and so G is the centre of gravity.

Thus the *C.G. of a thin uniform rod is at its middle point.*

2. C. G. of a thin plate or lamina in the form of a parallelogram.

Let ABCD be a lamina in the form of a parallelogram and let E,F,L,M be the midpoints of the sides AB, CD, AD and BC, respectively.

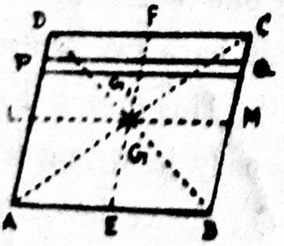


Fig. 5

Divide the lamina into a large number of elementary strips || to AB and let PQ be one such strip.

PQ can be considered to be a thin uniform rod and so its C.G. will be at G_1, its midpoint.

Clearly G_1 lies on EF.

C.G

Similarly, the C.G of each of the other strips lies on EF and so the C.G of the whole figure lies on EF.

Again by dividing the lamina into a large number of strips || to AD, we see that its C.G. must lie on the line LM which joins the midpoints of AD and BC.

Hence the centre of gravity is at G, the point of intersection of the lines EF and LM.

Clearly G is the point of intersection of the diagonals of the parallelogram.

Thus *the C.G. of a uniform parallelogram is at the point of intersection of its diagonals.*

§ 7. **Centre of Gravity by symmetry:** From § 6, it is clear that, if in a uniform body, we can find a point G such that the body can be divided into pairs of particles balancing about it, then G must be the C. G. of the body.

The C. G. of a uniform circle or uniform sphere, is therefore its centre.

If we can divide a lamina into strips, the C. G. of which all lie on a straight line, then the C. G. of the lamina must lie on that line.

Thus the positions of the centres of gravity of many bodies may be found by considerations of symmetry.

§ 8. C. G. of a uniform triangular lamina: very important

Let ABC be a uniform triangular lamina and D, E the midpoints of BC and CA respectively. Divide the area into a number of strips parallel to BC and let PQ be one such strip. Let AD meet PQ at L. As PQ is || to BC, the median AD bisects PQ also. i.e. L is the midpoint of PQ.

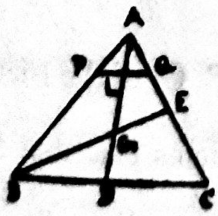


Fig. 6

So the C. G. of the strip PQ is at L i.e. it lies on AD.

Thus the C, G of each of the other parallel strips lies on AD. Hence the C. G. of the whole lamina lies on AD. Similarly by dividing the triangular area into strips parallel to AC, we find that the C. G lies on the median BE also.

Hence the C. G. of the triangle must be at G, the point of interesection of the medians, which is known as the centroid of the triangle. We know, from geometry, that the centroid divides each median in the ratio 2:1 from the vertex. Hence the C. G. of the triangle is the point G on the median AD such that AG:GD = 2:1.

IC from above theorem

§ 9. Theorem: *The centre of gravity of a uniform triangular lamina is the same as that of three equal particles placed at its vertices or at the middle points of its sides.*

Let ABC be a triangle and D,E,F the midpoints of its sides BC, CA AD respectively. The C. G. of the triangle is at the centroid G, which is on the median AD such that AG:GD = 2:1

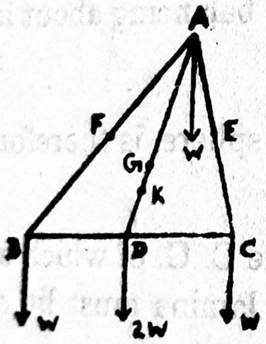


Fig 7.

Suppose three particles, each of weight W be placed at A, B, C. We shall find the C. G. of this particle system. The weight W at B and the weight W at C will give a resultant 2W acting at D, the midpoint of BC.

The weight 2W at D and the weight W at A will give a resultant 3W acting at the point K on DA such that

$$\frac{DK}{KA} = \frac{\text{weight at A}}{\text{weight at D}} = \frac{W}{2W} = \frac{1}{2}$$

i.e. K divides AD in the ratio 2:1.

So K must be the same point as G, the C. G. of the triangle.

Hence the C. G. of the three equal particles placed at A, B, C is the same as that of the lamina.

If equal weights are placed at the midpoints D, E, F, the C.G. of these weights must be at the centroid of the Δ DEF.

But, from geometry, we know that the centriod of the ΔDEF is the same point G, the centroid of ΔABC. Hence the C. G. of three equal particles placed at D, E, F also, will be the same as that of the triangle ABC.

Note: Let W be the weight of the whole triangle ABC. Instead of placing any three equal weights at the corners, suppose we place weights $\frac{W}{3}, \frac{W}{3}, \frac{W}{3}$ at A, B, C. In this case the C. G. of the particles is not altered. In addition, the total weight of the particles = W = the weight of the triangle.

Hence we have the following important proposition:

The weight of a triangle is statically equialent to that of three equal particles, each of one-third the total weight, placed at the vertices or at the midpoints of the sides.

§ 10. C. G. of three rods forming a triangle:

(or C. G. of a uniform wire bent into the form of a triangle)

Let ABC be the triangle formed by three uniform rods of the same thickness and material.

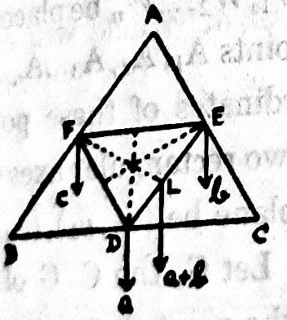


Fig. 8

Let D, E, F be the midpoints of BC, CA, AB.

Join DE, EF and FD.

The weights of the rods are proportional to their lengths a, b, c and they act at D, E, F.

The weight a at D and the weight b at E will give a resultant weight (a+b) acting at the point L on DE such that

$$\frac{DL}{LE} = \frac{\text{weight at E}}{\text{weight at D}} = \frac{b}{a} \quad \dots \dots (1)$$

But b = AC = 2 DF and a = BC = 2FE.

$$\therefore \frac{b}{a} = \frac{DF}{FE} \quad \dots \dots (2)$$

From (1) and (2), $\frac{DL}{LE} = \frac{DF}{FE}$

$\therefore$ FL bisects the angle DFE.

Now the weight (a+b) acting at L and the remaining weight c at F will gie a final resultant acting at some point on FL.

Hence the C. G. of the three rods must lie on FL, the bisector of $\angle$DFE. Similarly, by changing the order of compounding the weights a, b, c it clear that the C. G. of the rods must lie on the bisectors of $\angle$FDE and $\angle$FED also.

Thus the C. G. of the rods is the point of intersection of the internal bisectors of the angles of the triangle DEF. i.e. it is at the incentre of the ΔDEF.

§ 11. General formulae for determination of the G. G:

We will now obtain formulare for determining the C. G. of any system of particles, whose positions and weights are known.

Let a number of particles of weights $W_1, W_2, \ldots W_n$ be placed in a plane at points $A_1, A_2, A_3 \ldots A_n$ and let the coordinates of these points referred to two rectangular axes OX, OY in the plane be $(x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$. Let G, the C G of the system, be the point $(\bar{x}, \bar{y})$,

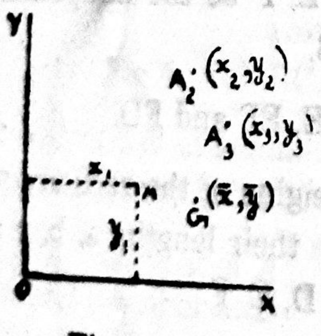


Fig. 9

Now the position of G relative to the plane of the particles does not depend on the position of the plane.

Hence to find $\bar{x}$, we can assume that the plane is vertical and is so placed that the x - axis is horizontal. Hence the weights W_1, W_2, W_3 etc, will be all like parallel forces, parallel to the y - axis The resultant of the weights is a force R = $W_1 + W_2 + W_3 + \ldots$ which is also parallel to the y - axis, acting through the point G $(\bar{x}, \bar{y})$.

Hence taking moment about O, we have

$$R.\ \bar{x} = W_1x_1 + W_2x_2 + W_3x_3 + \ldots$$

$$\text{i.e. } \bar{x} = \frac{W_1x_1 + W_2x_2 + W_3x_3 + \ldots\ldots}{W_1 + W_2 + W_3 + \ldots.} = \frac{\Sigma W x_i}{\Sigma W} \ldots. (1)$$

For finding $\bar{y}$, we assume that the plane is vertical and is so placed that the y axis is horizontal. The weights W_1, W_2 etc. will be all like parallel forces, parallel to the x - axis.

Taking moments about G, we get

$$\text{i.e. } \bar{y} = \frac{W_1y_1 + W_2y_2 + W_3y_3 + \ldots}{W_1 + W_2 + W_3 + \ldots.} = \frac{\sum Wy}{\sum W} \ldots\ldots (2)$$

Equations (1) and (2) gie the coordinates of G.

Note: 1. Since W = mg. where m is the mass of the particles, the above formulae can also be written as

$$\bar{x} = \frac{\Sigma mx}{\Sigma m} \text{ and } \bar{y} = \frac{\Sigma my}{\Sigma m}$$

The point thus found by considering the masses of the particles instead of their weights is the *Centre of mass*. The centre of mass and the centre of gravity are usually considered to be the same.

2. In the above result, $\bar{x}$ is the distance of the C. G. of the system of particles from OY and $\bar{y}$ is the distance of the C. G. from the line OX. As OX and OY can be any two lines in the plane, we can generalise the above result as follows:

Let $p_1, p_2, \ldots p_n$ be the distances of the particles of weights $w_1 w_2 \ldots w_n$ from any line in their plane and p the distance of their C. G. from the same line.

Then $p = \dfrac{w_1 p_1 + w_2 p_2 + \dots\dots + w_n p_n}{w_1 + w_2 + \dots\dots + w_n}$

3. If $P_1, P_2, P_3 \dots \dots$ are a set of like parallel forces acting at points $(x_1, y_1)(x_2 y_2)$ etc., then $(\bar{x}, \bar{y})$ the centre of parallel forces is given by

$$\bar{x} = \frac{P_1 x_1 + P_2 x_2 + \dots\dots}{P_1 + P_2 + \dots\dots}$$

$$\bar{y} = \frac{P_1 y_1 + P_2 y_2 + \dots\dots}{P_1 + P_2 + \dots\dots}$$

WORKED EXAMPLES

5M Important problem

Ex. 1. *A thin wire is bent into the form of a triangle ABC and heavy particles of weights P, Q, R are placed at the angular points. Prove that, if the centre of mass of the particles concides with that of the wire, then*

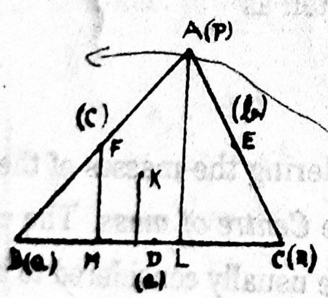
Friday

$$\frac{P}{b+c} = \frac{Q}{c+a} = \frac{R}{a+b}$$

Let D, E, F the midpoints of the sides BC, CA, AB respectively. Draw AL and FM $\perp$ to BC.

P Q R

AL . 0 . 0



Let K be the C. G. of the masses P, Q, R placed at the vertices.

The distances of P Q, R from BC are AL, 0 and 0.

If α is the distance of K from BC, then

Fig. 10

$$\alpha = \frac{P.AL + Q.0 + R.0}{P + Q + R} \quad \text{(Refer note 2 on page 279)}$$

$$= \frac{P.AL}{P + Q + R} \dots\dots\dots (1)$$

Consider now the wire, bent in the form of $\triangle$ ABC. This can be divided into three portions BC, CA and AB, mass of each being proportional to the lengths a, b, c respectively.

We therefore have three masses namely a at D, b at E and c at F.

The distances of masses b and c from BC are FM and the distance of the mass a from BC = 0.

Since K is the C. G. of the wire also, we have

$$\alpha = \frac{a \times 0 + b.\text{FM} + c.\text{FM}}{a+b+c}$$

b c a

FM FM 0

$$= \frac{(b+c)\text{FM}}{(a+b+c)} = \frac{(b+c)}{(a+b+c)}\frac{1}{2}.\text{AL} \quad \ldots \ldots (2)$$

FM = 1/2 AL

Equating (1) and (2).

$$\frac{\text{P.AL}}{\text{P} + \text{Q} + \text{R}} = (b+c)\tfrac{1}{2}.\frac{\text{AL}}{(a+b+c)}$$

i.e. $$\frac{2\text{P}}{\text{P}+\text{Q}+\text{R}} = \frac{b+c}{a+b+c}$$

or $$\frac{\text{P}}{b+c} = \frac{\text{P+Q+R}}{2(a+b+c)} \quad \ldots \ldots \ldots (3)$$

Similarly by considering the perpendicular distances of the point K from the sides CA and AB by two methods in each case, we will have

$$\frac{\text{Q}}{c+a} = \frac{\text{P}+\text{Q}+\text{R}}{2(a+b+c)} \ldots \ldots \ldots (4)$$

and $$\frac{\text{R}}{a+b} = \frac{\text{P}+\text{Q}+\text{R}}{2(a+b+c)} \ldots \ldots \ldots (5)$$

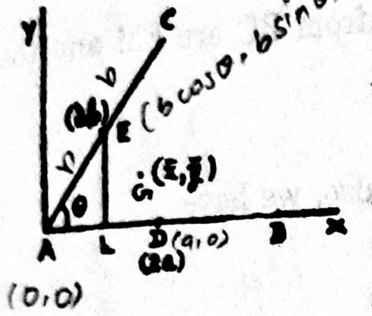
∴ Equating the left side members in (3), (4) and (5).

we get $\frac{\text{P}}{a+c} = \frac{\text{Q}}{c+a} = \frac{\text{R}}{a+b}$.

✓ **Ex. 2.** *AB and AC are two uniform rods of lengths 2a and 2b respectiely. If ∠BAC = θ, proe that the distance from A of the C. G. of the two rods is* $\frac{(a^4 + 2a^2b^2\cos\theta + b^4)^{\frac{1}{2}}}{a+b}$

problems

Let D and E be the midpoints of AB and AC.



The masses of the rods are proportional to their lengths. Take AB as x axis and the line through A, ⊥ to AB as the y axis. Draw EL ⊥ to AB.

Then AL = AE $\cos\theta = b\cos\theta$ and EL $= b\sin\theta$. The coordinates of E are $(b\cos\theta, b\sin\theta)$ and those of D are (a, 0).

Fig. 11

We have therefore to find the C. G. of the two masses namely one mass 2b at E and the other mass 2a at D.

Let G $(\bar{x}, \bar{y})$ be the coordinates of their C. G. ~~Applying the formula of § 11~~,

$$\bar{x} = \frac{2b.b\cos\theta + 2a.a}{2b + 2a} = \frac{a^2 + b^2\cos\theta}{a+b}$$

$$\bar{y} = \frac{2b.b\sin\theta + 2a.o}{2b + 2a} = \frac{b^2\sin\theta}{a+b}$$

A is(0, 0).

$$\therefore AG^2 = \bar{x}^2 + \bar{y}^2$$

$$= \left(\frac{a^2 + b^2\cos\theta}{a+b}\right)^2 + \left(\frac{b^2\sin\theta}{a+b}\right)^2$$

$$= \frac{a^4 + 2a^2b^2\cos\theta + b^4\cos^2\theta + b^4\sin^2\theta}{(a+b)^2}$$

$$= \frac{a^4 + 2a^2b^2\cos\theta + b^4}{(a+b)^2}$$

$$\therefore AG = (a^4 + 2a^2b^2\cos\theta + b^4)^{\frac{1}{2}} \div (a+b)$$

Ex. 3. *A wire of length 5a is bent so as to form the five sides of a regular hexagon. Show that the distance of its centre of gravity from either end of the wire is* $\frac{a}{10}\sqrt{133}$

(B.Sc. 77)

Let AFEDCB be the wire forming the regular hexagon ABCDEF. It can be divided into five portions with centres of gravity at their midpoints G_1, G_2, G_3, G_4, G_5 as shown in the figure. The weight of each portion is proportional to a, its length.

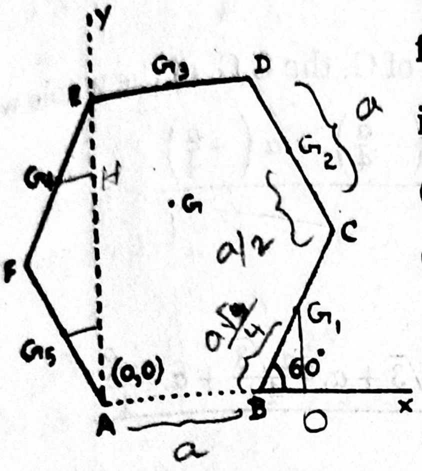


Fig 12.

Take AB and AE as x and y axes.

$\sin 60° = \frac{OG_1}{BG_1}$

$OG_1 = BG_1 \sin 60°$

y coordinate of G_1 and G_5

$= BG_1 . \sin 60° = \frac{a}{2} . \frac{\sqrt{3}}{2} = \frac{a\sqrt{3}}{4}$

y coordinate of G_2 and G_4 $= \frac{3a\sqrt{3}}{4}$

y coordinate of $G_3 = 4\frac{a\sqrt{3}}{4} = a\sqrt{3}$

$\frac{a}{2} + \frac{a\sqrt{3}}{4} = \frac{3a\sqrt{3}}{4}$

x coordinate of G_1 and G_2 $= AB + G_1B . \cos 60°$

$= a + \frac{a}{2} \cos 60$

$= a + \frac{a}{2} \times \frac{1}{2} = \frac{5a}{4}$

x coordinat $= AB + BO$

$= a + a/4$

$= 5a/4$

x coordinate of $G_3 = \frac{a}{2}$

and x coordinate of G_5 and $G_4 = -\frac{a}{4}$

$\cos 60° = \frac{G_4H}{EG_4}$

$EG_4 \cos 60° = HG_4$

We can conveniently tabulate the masses acting at G_1, G_2, etc. and the coordinates as follows:

Mass	x-coordinate	y-coordinate
$a(G_1)$	$\frac{5a}{4}$	$\frac{a\sqrt{3}}{4}$
$a(G_2)$	$\frac{5a}{4}$	$\frac{3a\sqrt{3}}{4}$
$a(G_3)$	$\frac{a}{2}$	$a\sqrt{3}$

$a(G_4)$	$-\frac{a}{4}$	$\frac{3a\sqrt{3}}{4}$
$a(G_5)$	$-\frac{a}{4}$	$\frac{a\sqrt{3}}{4}$

Let $(\bar{x}, \bar{y})$ be the coordinates of G, the C.G. of the whole wire.

$$\bar{x} = \frac{a.\frac{5a}{4} + a.\frac{5a}{4} + a.\frac{a}{2} + a\left(-\frac{a}{4}\right) + a\left(-\frac{a}{4}\right)}{5a}$$

$$= \frac{5a^2}{2} \times \frac{1}{5a} = \frac{a}{2}$$

$$\bar{y} = \frac{a.\frac{a\sqrt{3}}{4} + a.\frac{3a\sqrt{3}}{4} + a.a\sqrt{3} + a.\frac{3a\sqrt{3}}{4} + a.\frac{a\sqrt{3}}{4}}{5a}$$

$$= \frac{3a^2\sqrt{3}}{5a} = \frac{3a\sqrt{3}}{5}$$

$\therefore$ G is $\left(\frac{a}{2}, \frac{3a\sqrt{3}}{5}\right)$

$AG = \sqrt{(x_2 - x_1)^2 + (y_2 -}$

A is (0, 0) and B is (a, 0)

$AG^2 = (x_2 - x_1)^2 + ($

$$\therefore AG^2 = \left(\frac{a}{2} - 0\right)^2 + \left(\frac{3a\sqrt{3}}{5} - 0\right)^2$$

Same Important Sum

$$= \frac{a^2}{4} + \frac{27a^2}{25} = \frac{133a^2}{100}$$

$$\therefore AG = \frac{a\sqrt{133}}{10}$$

$$BG^2 = \left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{3a\sqrt{3}}{2}\right)^2 = \frac{133a^2}{100}$$

Hence BG is also $= \frac{a\sqrt{133}}{10}$

Ex. 5. *ABCD is a trapezium in which AB and CD are parallel and of lengths a and b. Prove that the distance of the centre of mass from AB is* $\frac{h}{3}\left(\frac{a+2b}{a+b}\right)$ *where h is the distance between AB and CD.*

Sunday

(B.Sc. 54; B.A. 50, 45)

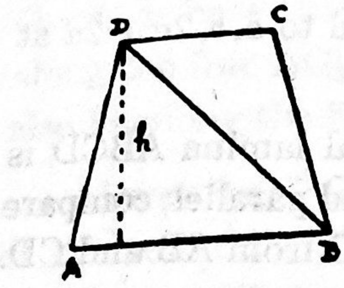


Fig.15.

Join BD. The trapezium is divided into two triangles ADB and BCD.

$\Delta ADB = \frac{1}{2}AB.h = \frac{1}{2}ah$ and

$\Delta BCD = \frac{1}{2}CD.h = \frac{1}{2}bh.$

Replace the weight of each of these triangles by particles equal to one-third of their weights, placed at their angular points. We then have the following distribution.

Point	weight	Distance from AB
A	$\frac{ah}{6}$	0
D	$\frac{ah}{6}+\frac{bh}{6}$	h
B	$\frac{ah}{6}+\frac{bh}{6}$	0
C	$\frac{bh}{6}$	h

If r is the distance of the C. G. from AB,

$$r = \frac{\frac{ah}{6}\times 0+\left(\frac{ah}{6}+\frac{bh}{6}\right)h+\left(\frac{ah}{6}+\frac{bh}{6}\right)0+\frac{bh}{6}\times h}{\frac{ah}{6}+\frac{ah}{6}+\frac{bh}{6}+\frac{ah}{6}+\frac{bh}{6}+\frac{bh}{6}}$$

$$=\frac{ah^2+2bh^2}{3(ah+bh)}=\frac{h(a+2b)}{3(a+b)}$$

Finite